Parity and spin of the Θ^+ pentaquark in the $NN \to Y\Theta^+$ reaction at the threshold

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Abstract

Spin observables of a binary reaction $1 + 2 \to 3 + 4$ are discussed at the threshold in general form using P-parity and angular momentum conservation. General formulae for polarization transfer, spin-spin correlation parameters in the initial and final states and induced tensor polarization are derived for arbitrary spins of participating particles. This formalism is worked out in detail for the $NN \to Y\Theta^+$ reaction. Assuming that the spin of the pentaquark Θ^+ takes the values $\frac{1}{2}$, $\frac{3}{2}$ and $\frac{5}{2}$, whereas the spin of the hyperon Y equals $\frac{1}{2}$, explicite formulae are obtained for the observables in terms of few non-vanishing at the threshold spin amplitudes separately for the spin-singlet and spin-triplet initial NN states. In case of all particles in the $NN \to Y\Theta^+$ reaction have the spin-$\frac{1}{2}$ a full spin-structure for totally polarized cross sections is derived. Some of the obtained spin observables strongly depend on the intrinsic P-parity of the Θ^+ and the total isospin of the reaction. Therefore, measurements of these observables allow one to determine the P-parity of the pentaquark Θ^+ in a model independent way for any spin of the Θ^+.

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1 Introduction

Experimental indications [1,2,3,4,5,6,7,8,9,10] to existence of the exotic baryon $\Theta^+(1540)$ with the strangeness $S = +1$ stimulated an intensive theoretical discussion during the past year. A minimal number of constituent quarks in the $\Theta^+(1540)$, compatible with its positive strangeness and the baryon charge, is five and corresponds to the quark content $uudd\bar{s}$. The structure of the $\Theta^+(1540)$ and its quantum numbers originally predicted in the chiral soliton model [11] ($j^\pi_\Theta = \frac{1}{2}^+$), have been considered yet in several quark models with a different type of quark-quark interaction, as well as within the QCD sum rules and Lattice QCD. For recent review see, for example, Refs. [12]. A surprisingly narrow width of the Θ^+ ($\Gamma_\Theta \approx 15$ MeV) predicted by the chiral soliton model [11], now is expected to be about of 1 MeV or less according to the recent analysis [13] of the KN- and Kd- scattering data and the DIANA data [2] on the Θ^+ production [14]. This looks also as an exotic itself in view of the open decay channel $\Theta^+ \to KN$ with the same quark content in the KN system as in the Θ^+. The width of the Θ^+ is now a challenge for any theoretical approach [15,16,17,18,19,20], including the chiral soliton model also [21,22]. The intrinsic parity of the Θ^+ is of particular importance to understand the nature of this baryon. A positive parity of the Θ^+ is predicted by the chiral soliton model [11]. On the contrary, naive quark shell models give the negative parity for the S-wave ground state of the quark system $uudd\bar{s}$. Quark-quark interaction could lead to a P-wave state and, hence, to positive parity of the Θ^+ [23,24,25]. In Lattice QCD, both the positive [25] and negative [26] parity of the Θ^+ are obtained although in Refs. [27] the $\Theta^+(1540)$ resonance state is not found. A possible correlation between the positive parity of the Θ^+ and its narrow width is discussed in [28] within a constituent quark model. Experimental determination of the quantum numbers of the Θ^+ will be essential for establishing of the underlying dynamics of the Θ^+. [2]

[2] Besides of the $\Theta^+(1540)$ observation, an indication to existence of two others explicitely exotic baryons were also reported: $\Xi^{--}(1860)$ [29] and $\Theta^0_c(3099)$ [30]. However, in several experiments performed mainly at high energies the states $\Theta^+(1540)$,

Several methods, based on certain assumptions on the production mechanism, were suggested for determination of the P-parity of the Θ^+, π_Θ, in Kp-scattering [34] and photoproduction reactions [35]. Model independent methods were also considered [36]. We discuss here the Θ^+ production in NN-collision. Model independent methods for determination of the P-parity of the Θ^+ in the reaction $NN \to Y\,\Theta^+$ were suggested in Refs. [37,38,39,40]. These methods are based on such general properties of the reaction amplitude as angular momentum and P-parity conservation and on the generalized Pauli principle for nucleons. Assuming for the spin of the Θ^+ the value $j_\Theta = \frac{1}{2}$, it was shown in Ref.[38] that the sign of the spin-spin correlation parameter $C_{y,y}$ unambiguously determines the P-parity of the Θ^+ in the reaction $pp \to \Sigma^+\Theta^+$. Another definite correlation between $C_{y,y}$ and π_Θ holds for the $pn \to \Lambda^0\Theta^+$ reaction [39,40] if the isospin of the Θ^+ equals zero. Furthermore, a measurement of the spin transfer coefficients $K_y^y = K_x^x$ or K_z^z in these reactions also allows to determine the P-parity unambiguously [39,40]. Obviously, a double-spin measurement near the threshold is a challenge for experiment. In this connection, one should note that measurements of the polarization transfer from the initial nucleon to the hyperon in the reaction $NN \to Y\Theta^+$ can be performed by single spin experiments with only polarized beam or target, since the polarization of the hyperon can be measured via its weak decay.

The results of Refs. [37,38,40] are based on the assumption that the spin of the Θ^+ is equal to $\frac{1}{2}$. Up to now the experimental value of the spin of the Θ^+ is not known, as well as its P-parity. Analysis of the angular distribution in the system nK^+ in the final state of the reaction $np \to npK^+K^-$ measured in [10], leads the authors of Ref.[10] to a conclusion that the minimal value of the spin of the $\Theta^+(1540)$ is $\frac{5}{2}$. Furthermore, within some quark models the

$\Xi^{--}(1860)$, $\Theta_c^0(3099)$ were not found. For review of positive and null experimental results one can see Refs. [31,32]. At present, null results obtained at high energies do not mean that the experiments [1,2,3,4,5,6,7,8,9,10] are incorrect. Most likely, the $\Theta^+(1540)$ production mechanism depends on energy and transferred momentum in such a way that at high energies this state is non-visible [33]. New experiments with high statistics are planned [32] to get definite answer about existence of the exotic baryons.

pentaquark being the spin-$\frac{1}{2}$ baryon has partners with the spin $\frac{3}{2}$. According to Ref. [41], the spin-$\frac{3}{2}$ partner can have a mass very close to the Θ^+. As was noted recently in Ref. [17], the $(0s)^5$ configuration of the $uudd\bar{s}$ pentaquark being in the $\frac{3}{2}^-$ state cannot decay into a KN d-wave state. Hence the $\frac{3}{2}^-$ state is a candidate for the observed narrow $\Theta(1540)$ resonance. A possibility that Θ^+ is the $\frac{3}{2}$-spin state was discussed also in Refs. [18,20,42,43]. At last, higher spins are allowed for the pentaquarks Θ^* in higher SU(3) multiplets.

Therefore a generalization of the approach [37,38] to arbitrary spin of the Θ^+ is an important task. This problem was discussed in Ref. [45]. For arbitrary spin of the Θ^+, it was shown in [45] that the signals for determination of the P-parity of the Θ^+ via measurements of the $C_{i,j}$ and K_i^j in the reaction $NN \to Y\Theta^+$ are the same as for the case of the minimal spin $j_\Theta = \frac{1}{2}$ [3].

In this work we develop the method of Refs. [39,45] for calculation of $C_{i,j}$ and K_i^j for arbitrary spins of *all* participating particles in the binary reaction $1 + 2 \to 3 + 4$. Furthermore, we obtain formulae for the spin-spin correlation parameters in the final state of the binary reaction. In this case beam and target are assumed to be unpolarized. For the case of $j_\Theta > \frac{1}{2}$ we consider also the tensor polarization of the Θ^+, t_{J0}, both for polarized and unpolarized beam. For even rank J, tensor polarization can be measured via analysis of angular distribution in the strong decay $\Theta^+ \to N + K$. We show that t_{J0} induced by polarized beam depends on the P-parity of the Θ^+. The derived general formulae can be applied for *any binary reaction* near threshold. For the $NN \to Y\Theta^+$ reaction we present detailed formulae assuming $j_\Theta = \frac{1}{2}, \frac{3}{2}$ and $\frac{5}{2}$. In case of all participants in the $NN \to Y\Theta^+$ reaction are the spin-$\frac{1}{2}$ particles we derive totally polarized cross sections for the isospin $T = 0$ and $T = 1$ of the NN channel, taking into account all polarizations in the initial and final states. Analysis is based on common properties of the reaction amplitude and the standard technique of the spin-tensor operators [46]. For the

[3] Independently this problem was considered in Ref. [47] in the $\boldsymbol{\sigma}$- representation for the transition amplitude.

case of all particles are the spin-$\frac{1}{2}$ baryons we apply also the $\boldsymbol{\sigma}$- representation to compare the results with a general method. In addition, we develop a non-standard method, which allows us to derive an elegant formula for $C_{i,j}$ for the binary reaction with initial particles being of spin-$\frac{1}{2}$ and final particles of arbitrary spins.

The paper is organized in the following way. In section 2 the partial wave expansion is given for the transition amplitude near the threshold. The spin-spin correlation parameters in the initial state of the reaction $NN \to Y\,\Theta^+$ are discussed in section 3 using a non-standard method. In section 4 a general method is elaborated to derive formulae for the spin-transfer coefficients, tensor polarization induced by polarized or non-polarized beam, and spin-spin correlation coefficients for arbitrary spins in the binary reaction $1+2 \to 3+4$. Explicite formulae are given for different values of the Θ^+ spin. In section 5 we derive the full spin structure of the cross section of the reaction $NN \to Y\Theta^+$ for the case of the spin-$\frac{1}{2}$ particles. The results are discussed in section 6. Some technical details are explained in Appendix.

2 Transition amplitude

Assuming dominance of the s-wave in the relative motion in the final system, the most general expression for the amplitude of the binary reaction $1+2 \to 3+4$ at the threshold can be written as [48]

$$T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2} = \sum_{\substack{J\,M \\ S\,M_S\,L\,m}} (j_1\mu_1\,j_2\mu_2|SM_S)(j_3\mu_3\,j_4\,\mu_4|J\,M) \times \\ \times (S\,M_S\,L\,m|J\,M) Y_{Lm}(\hat{\mathbf{k}}) a^{LS}_J. \tag{1}$$

Here j_i and μ_i are the spin of the i-th particle and its z-projection, J and M are the total angular momentum and its z-projection; S and L are the spin and orbital momentum of the initial system, respectively, and M_S and m are the corresponding z-projections. The coupling scheme for angular momenta is

given by

$$\hat{\mathbf{S}} = \hat{\mathbf{j}}(1) + \hat{\mathbf{j}}(2), \;\; \hat{\mathbf{S}} + \hat{\mathbf{L}} = \hat{\mathbf{J}}, \;\; \hat{\mathbf{j}}(3) + \hat{\mathbf{j}}(4) = \hat{\mathbf{J}}, \tag{2}$$

where $\hat{\mathbf{j}}(i)$ is the spin-operator of the i-th particle. Information on the reaction dynamics is contained in the complex amplitudes $a_J^{L\,S}$. The sum over J in Eq. (1) is restricted by the conditions $J = j_3 + j_4, j_3 + j_4 - 1, ..., |j_3 - j_4|$. We choose the z-axis along the unit vector of the initial momentum $\hat{\mathbf{k}}$, therefore $Y_{Lm}(\hat{\mathbf{k}}) = \sqrt{(2\,L+1)/4\pi}\,\delta_{m\,0}$. Due to P-parity conservation, the orbital momentum L in Eq. (1) is restricted by the condition $(-1)^L = \pi$, where $\pi = \pi_1\,\pi_2\,\pi_3\pi_4$, and π_i is the intrinsic P-parity of the i-th particle. We consider here mainly transitions without mixing the total isospin T in this reaction [4]. Using the generalized Pauli principle, one can find that for given values of T and π the spin of the initial nucleons S is fixed unambiguously by the following relation

$$(-1)^S = \pi(-1)^{T+1}. \tag{3}$$

Therefore, in order to determine the P-parity π of the system at a given isospin T, it is sufficient to determine the spin of the NN-system in the initial state of this reaction.

Let us to determine the number of the spin amplitudes a_J^{LS} for a particular case of $j_3 = \frac{1}{2}$ and j_4 being half-integer, $j_4 = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$. For this case there are two total angular momenta $J_p = j_4 + \frac{1}{2}$ and $J_m = j_4 - \frac{1}{2}$. For the spin-singlet initial state $S = 0$ only one orbital momentum is allowed, $L = J$, and therefore there is only one scalar amplitude, $a_J^{L\,S} = a_J^{J\,0}$, where $(-1)^L = \pi$. For $S = 1$ and $j_4 \geq \frac{3}{2}$ there are three scalar amplitudes $a_J^{L\,1} \equiv a_J^L$:

(i) $a_{J_p}^{J_p}$, $a_{J_m}^{J_m+1}$ and $a_{J_m}^{J_m-1}$, if $(-1)^{J_p} = \pi$,

or

(ii) $a_{J_m}^{J_m}$ $(J_m \neq 0)$, $a_{J_p}^{J_p+1}$ and $a_{J_p}^{J_p-1}$, if $(-1)^{J_p} = -\pi$.

For particular case of $j_4 = j_3 = \frac{1}{2}$, one has $J_m = 0$ and $J_p = 1$. For this case

[4] The isospin mixing is possible, for example, in the reaction $p + n \to \Sigma^0 + \Theta^+$, if the Θ^+ is an isotriplet. In this case the P-parity cannot be determined by using the method in question.

only two triplet amplitudes are allowed for $\pi = +1$, i.e. a_1^0 and a_1^2, whereas the amplitude a_0^0 is forbidden by conservation of the total angular momentum. For $\pi = -1$ one has also only two spin-triplet amplitudes, one of them corresponds to $J = 1$, a_1^1, and another one is allowed for $J = 0$, i.e. a_0^1.

3 Non-standard method

For $j_1 = j_2 = \frac{1}{2}$ and for given spin S, one can find from Eq. (1) the polarized cross section $d\sigma(\mathbf{p}_1, \mathbf{p}_2)$ as

$$d\sigma(\mathbf{p}_1, \mathbf{p}_2) = \Phi \sum_{\mu_3\,\mu_4} |T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2}|^2 =$$
$$= \frac{1}{4\pi} \sum_M (\frac{1}{2}\mu_1 \frac{1}{2}\mu_2|SM)^2 \sum_{J\,M\,L\,L'} \sqrt{(2L+1)(2L'+1)}(S\,M\,L\,0|J\,M) \times$$
$$\times (S\,M\,L'\,0|J\,M)\, a_J^{LS}\, (a_J^{L'S})^*, \quad (4)$$

where Φ is a kinematical factor. Using the relations $(\frac{1}{2}\mu_1\frac{1}{2}\mu_2|00) = \chi^+_{\mu_1}\frac{i\sigma_y}{\sqrt{2}}\chi^{(T)+}_{\mu_2}$ and $(\frac{1}{2}\mu_1\frac{1}{2}\mu_2|1\lambda) = \chi^+_{\mu_1}\sigma_\lambda\frac{i\sigma_y}{\sqrt{2}}\chi^{(T)+}_{\mu_2}$, where σ_i $(i = y, \lambda)$ is the Pauli matrix and χ_μ is the 2-spinor, one can find

$$(\frac{1}{2}\mu_1\frac{1}{2}\mu_2|00)^2 = \frac{1}{4}(1 - \mathbf{p}_1 \cdot \mathbf{p}_2), \quad (5)$$

$$(\frac{1}{2}\mu_1\frac{1}{2}\mu_2|1M)^2 = \begin{cases} \frac{1}{4}(1 + \mathbf{p}_1 \cdot \mathbf{p}_2 - 2p_{1z}p_{2z}), \; M = 0, \\ \frac{1}{4}[1 \pm (p_{1z} + p_{2z}) + p_{1z}p_{2z}], \; M = \pm 1. \end{cases} \quad (6)$$

In Eqs. (4), (5) and (6) $\mathbf{p}_i$ is the polarization vector of the i-th particle with the spin $j_i = \frac{1}{2}$ being in the pure spin state χ_{μ_i}. [5] The unpolarized cross section is given as

$$d\sigma_0 = \Phi\frac{1}{4}\sum_{\mu_1\,\mu_2\,\mu_3\,\mu_4} |T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2}|^2 = \frac{1}{16\,\pi}\Phi\sum_{J,L}(2J+1)|a_J^{L\,S}|^2. \quad (7)$$

[5] On the other side, Eqs. (5) and (6) can be considered as the matrix elements of the projection operator $|SM >< SM|$ [49] taken between the states $|\frac{1}{2}\mu_1, \frac{1}{2}\mu_2 >$.

3.1 *The spin-singlet initial state.*

Using Eqs. (4), (5) and (7) one can find for the spin-singlet polarized cross section the following formula

$$d\sigma(\mathbf{p}_1, \mathbf{p}_2) = d\sigma_0(1 - \mathbf{p}_1 \cdot \mathbf{p}_2). \tag{8}$$

As seen from this formula, the spin-singlet cross section is equal to zero, if the polarization vectors of colliding particles are parallel ($\mathbf{p}_1 \uparrow\uparrow \mathbf{p}_2$) and have maximal values ($|\mathbf{p}_1| = |\mathbf{p}_2| = 1$). In notations of Ref. [50], non-zero spin-spin correlation parameters for this case are the following:

$$C_{x,x} = C_{y,y} = C_{z,z} = -1, \text{ if } S = 0. \tag{9}$$

In order to find spin-transfer coefficients, one should consider the following cross section

$$d\sigma(\mathbf{p}_1, \mathbf{p}_3) = \Phi \sum_{\mu_2,\mu_4} |T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2}|^2. \tag{10}$$

The polarization vector $\mathbf{p}_1$ of the *1*-st particle in the right side of Eq.(10) can be found only in the following sum

$$\sum_{\mu_2} (\frac{1}{2}\mu_1 \frac{1}{2}\mu_2|00)^2 = \frac{1}{2}\sum_{\mu_2}(\chi^+_{\mu_1} i\sigma_y \chi^{(T)+}_{\mu_2})(\chi^T_{\mu_2}(-i\sigma_y)\chi_{\mu_1}) =$$
$$= \frac{1}{4} Sp(1 + \boldsymbol{\sigma}\cdot\mathbf{p}_1) = \frac{1}{2}. \tag{11}$$

Since the vector $\mathbf{p}_1$ is absent actually in the right hand side of Eq. (11), one should conclude that the all polarization transfer coefficients are zero for the spin-singlet initial state: $K_i^j = 0\,(i, j = x, y, z)$. The obtained results for $C_{i,j}$ and K_i^j are valid for any values of the spins j_3 and j_4, both of them being integer or half-integer.

3.2 *The spin-triplet initial state.*

For $S = 1$ and $M = 0$, Eq. (4) can be written as

$$d\sigma_{M=0}(\mathbf{p}_1,\mathbf{p}_2) = \frac{\Phi}{16\,\pi}(1+\mathbf{p}_1\cdot\mathbf{p}_2 - 2p_{1z}p_{2z})\ \times$$
$$\sum_J |\sqrt{J}\,a_J^{J-1} - \sqrt{J+1}\,a_J^{J+1}|^2. \tag{12}$$

We obtain this formula from Eq. (4) using Eq. (6) and the following formulae for the Clebsch-Gordan coefficients: $(1\,0\,J\,0|J\,0) = 0, (1\,0\,J-1\,0|J\,0) = \sqrt{J/(2J-1)}$, $(1\,0\,J+1\,0|J\,0) = -\sqrt{(J+1)/(2J+3)}$. In order to simplify the notations, we omit in Eq. (12) and below the superscript $S=1$ in $a_J^{L\,S}$. The sum over the projections $M=+1$ and $M=-1$ into right-hand side of Eq. (4) gives

$$d\sigma_{M=\pm1}(\mathbf{p}_1,\mathbf{p}_2) = \frac{\Phi}{16\,\pi}(1+p_{1z}p_{2z})\times$$
$$\times\begin{cases}\sum_J |\sqrt{J}\,a_J^{J+1} + \sqrt{J+1}\,a_J^{J-1}|^2, & \text{if}\quad (-1)^{J+1}=\pi,\\ \sum_J (2J+1)\,|a_J^J|^2, & \text{if}\ \ (-1)^J = \pi.\end{cases} \tag{13}$$

Here we used the following relations: $(1 1\,J-1|J\,0) = \frac{1}{\sqrt{2}}$, $(1 1\,J-1|J-1\,0) = \sqrt{J+1}/\sqrt{2(2J+1)}$, $(1 1\,J\ -1|J+1\,0) = \sqrt{J}/\sqrt{2(2J+1)}$. Using Eqs.(12), (13), one can make summation over M in Eq. (4) and then present the spin-triplet polarized cross section in the following standard form [50]

$$d\sigma(\mathbf{p}_1,\mathbf{p}_2) = d\sigma_0\,(1 + C_{x,x}\,p_{1x}\,p_{2x} + C_{y,y}\,p_{1y}p_{2y} + C_{z,z}\,p_{1z}\,p_{2z}), \tag{14}$$

where the spin-spin correlation parameters are given as

$$C_{x,x} = C_{y,y} = \frac{\sum_J |\sqrt{J}\,a_J^{J-1} - \sqrt{J+1}\,a_J^{J+1}|^2}{\sum_{J\,L}(2J+1)\,|a_J^L|^2}, \qquad \text{if } S=1, \tag{15}$$
$$C_{z,z} = 1 - 2\,C_{y,y}. \tag{16}$$

As seen from Eq. (15), the spin-spin correlation parameters are non-negative for transversal polarization. One can see from Eq. (15) that the diagonal term a_J^J does not contribute into the numerator of $C_{x,x}=C_{y,y}$. The obtained results for $C_{i,j}$ are valid for any values of the spins j_3 and j_4, both of them being integer or half-integer. Eqs. (15) and (16) are very simple for applications. Explicite formulae for different spins of the Θ^+ will be given in the next section.

Considering the sum $\sum_{\mu_2}(\frac{1}{2}\mu_1\frac{1}{2}\mu_2|1M)(\frac{1}{2}\mu_1\frac{1}{2}\mu_2|1M')$, one can find that this sum explicitly contains the polarization vector $\mathbf{p}_1$. Therefore, in contrast to the case of $S=0$, the spin-triplet initial state $S=1$ allows a non-zero polarization transfer in this reaction. In order to get the spin-transfer coefficients we use below a general method developed in Ref. [46].

4 General method

According to Ref. [46], the amplitude in Eq. (1) can be written as

$$T^{\mu_3\,\mu_4}_{\mu_1\,\mu_2} = \chi^{+}_{j_3\,\mu_3}\chi^{+}_{j_4\,\mu_4}\hat{F}\chi_{j_1\,\mu_1}\,\chi_{j_2\,\mu_2}, \tag{17}$$

where $\hat{F}$ is an operator acting on the spin states of the initial and final particles. This operator can be written as

$$\hat{F} = \sum_{m_1\,m_2\,m_3\,m_4} T^{m_3\,m_4}_{m_1\,m_2}\,\chi^{+}_{j_1\,m_1}(1)\chi^{+}_{j_2\,m_2}(2)\chi_{j_3\,m_3}(3)\,\chi_{j_4\,m_4}(4), \tag{18}$$

where $\chi_{j_k\,m_k}(k)$ is the spin function of the kth particle with the spin j_k and z-projection m_k and $T^{m_3\,m_4}_{m_1\,m_2}$ is defined by Eq. (1). The operator $\hat{F}$ is normalized to the unpolarized cross section as

$$d\sigma_0 = \frac{\Phi}{(2j_1+1)\,(2j_2+1)} SpFF^{+}. \tag{19}$$

For the spin-triplet case ($S=1$) with $j_1=j_2=j_3=\frac{1}{2}$ one can write

$$4\pi SpFF^{+} = \\ = \begin{cases} (2J_p+1)|a^{J_p}_{J_p}|^2 + (2J_m+1)\Big(|a^{J_m+1}_{J_m}|^2 + |a^{J_m-1}_{J_m}|^2\Big) & \text{if } (-1)^{J_p}=\pi, \\ (2J_m+1)|a^{J_m}_{J_m}|^2 + (2J_p+1)\Big(|a^{J_p+1}_{J_p}|^2 + |a^{J_p-1}_{J_p}|^2\Big) & \text{if } (-1)^{J_m}=\pi \end{cases} \tag{20}$$

for $j_4 > \frac{1}{2}$ and

$$4\pi SpFF^{+} = \begin{cases} 3|a^1_1|^2 + |a^1_0|^2 & \text{if } \pi=-1, \\ 3|a^2_1|^2 + 3|a^0_1|^2 & \text{if } \pi=+1 \end{cases} \tag{21}$$

for $j_4 = \frac{1}{2}$.

4.1 Spin- transfer coefficients.

The spin-transfer coefficient, describing the polarization transfer from the *1*st particle to the *3*-rd one in the reaction $\vec{j}_1 + j_2 \rightarrow \vec{j}_3 + j_4$, is given by the following formula [46]

$$K_\lambda^\kappa = \frac{SpF\,\hat{j}_\lambda(1)\,F^+\,\hat{j}_\kappa(3)}{SpF\,F^+\,j_1 j_3}, \tag{22}$$

where $\hat{j}_\lambda(1)$ and $\hat{j}_\kappa(3)$ are the spherical components of the spin operators of the particles 1 and 3 ($\lambda, \kappa = 0, \pm 1$), respectively. For arbitrary spins of all participating particles j_i ($i = 1, \ldots, 4$) we found from Eqs. (1), (17) and (22) the spin transfer coefficient in the following general form (for details, see Appendix):

$$\begin{aligned} K_\lambda^\kappa(1 \to 3)\,SpFF^+ = \delta_{\lambda,-\kappa}\,\frac{1}{4\pi}\,\sqrt{j_1^{-1} j_3^{-1}(j_1+1)(2j_1+1)(j_3+1)(2j_3+1)}\,\times \\ \times \sum_{S\,S'\,J\,J'\,L\,L'\,J_0} \sqrt{(2L+1)\,(2L'+1)}\,\times \\ \times\sqrt{(2S+1)\,(2S'+1)}(2J+1)\,(2J'+1)\,(-1)^{j_1+j_2+j_3+j_4+S'+J'+L+1}\,\times \\ \times(1\,-\lambda\,1\,\lambda|J_0\,0)(L'\,0\,L\,0\,|J_0 0)\,\times \\ \begin{Bmatrix} j_1 & j_2 & S \\ S' & 1 & j_1 \end{Bmatrix} \begin{Bmatrix} j_3 & j_4 & J' \\ J & 1 & j_3 \end{Bmatrix} \begin{Bmatrix} J & S & L \\ J' & S' & L' \\ 1 & 1 & J_0 \end{Bmatrix} a_J^{L\,S}(a_{J'}^{L'\,S'})^*. \end{aligned} \tag{23}$$

Here we used the standard notations for the 6j-and 9j-symbols [51]. In Eq. (23) the intermediate angular momentum J_0 is even: $J_0 = 0$ and 2. The value $J_0 = 1$ is excluded because the Clebsch-Gordan coefficient $(L'\,0\,L\,0\,|J_0\,0)$ equals zero for $L' + L + J_0 = odd$ and, furthermore, $L' + L$ is even due to P-parity conservation. Taking into account this fact, one can find from Eq.(23) the following relations (for details, see Appendix):

$$K_{+1}^{-1} = K_{-1}^{+1} = -K_x^x = -K_y^y,\; K_0^0 = K_z^z,\; K_z^x = K_z^y = K_x^z = K_y^z = 0, \tag{24}$$

and $K_i^j = 0$ at $i \neq j$, where $i, j = x, y, z$. From Eq. (23) we also find that there is no polarization transfer ($K_i^j = 0$, $i, j = x, y, z$) for $S = S' = 0$ in accordance with the discussion given in the section 3.1. These coefficients are also equal to zero for $J = J' = 0$:

$$K_i^j = 0 \;\; (i, j = x, y, z), \qquad \text{if } S = S' = 0 \text{ or } J = J' = 0. \tag{25}$$

For the spin-triplet transitions $S = S' = 1$, we find from Eq. (23) that $K_x^x = K_y^y \neq 0$ and $K_0^0 \equiv K_z^z \neq 0$. Eq. (23) is a generalization of the formula given by Eq.(5) in Ref. [45] for the particular case of $j_1 = j_3 = \frac{1}{2}$.

Eq. (23) is rather complicated due to its general applicability. For particular case of $j_1 = j_2 = j_3 = \frac{1}{2}$, we present below explicit formulae for K_i^j for $j_4 = \frac{1}{2}$, $\frac{3}{2}$ and $\frac{5}{2}$.

4.1.1 $j_4 = \frac{1}{2}$

For the total isospin $T = 0$ and the P-parity $\pi = +1$ one has $S = 1$. For this case Eq. (23) gives (using the notation $a_J^{L\,1} = a_J^L$)

$$K_y^y = \frac{|\sqrt{2}\, a_1^0 + a_1^2|^2 - 3\, Re\, (\sqrt{2}\, a_1^0 + a_1^2) a_1^{2*}}{3\, (|a_1^0|^2 + |a_1^2|^2)}, \tag{26}$$

$$K_z^z = \frac{|\sqrt{2}\, a_1^0 + a_1^2|^2}{3\, (|a_1^0|^2 + |a_1^2|^2)}. \tag{27}$$

For $T = 1$ and $\pi = -1$ one has $S = 1$. In this case Eq. (23) gives

$$K_y^y = \frac{\sqrt{6}\, Re\, a_0^1\, a_1^{1*}}{|a_0^1|^2 + 3|a_1^1|^2}, \tag{28}$$

$$K_z^z = \frac{3|a_1^1|^2}{|a_0^1|^2 + 3|a_1^1|^2}, \tag{29}$$

which coincide (except for notations) with those obtained recently in Ref. [40] in the $\boldsymbol{\sigma}$-representation for the amplitude.

4.1.2 $j_4 = \frac{3}{2}$

For higher spins of the 4th particle $j_4 \geq \frac{3}{2}$, Eq. (23) also gives non-zero coefficients K_x^x and K_z^z for $S = 1$. So, for $j_4 = \frac{3}{2}$ we find

$$4\pi\, SpFF^+\, K_y^y = 3|a_2^1|^2 - 3|a_2^3|^2 + Re\left(\sqrt{3}a_1^1\, a_2^{1^*} - \sqrt{\frac{9}{2}}a_1^1 a_2^{3^*} - \sqrt{\frac{3}{2}}a_2^3\, a_2^{1^*}\right),$$
$$4\pi\, SpFF^+\, K_z^z = \frac{3}{2}(|a_2^1|^2 - |a_1^1|^2) + |a_2^3|^2 + Re\left(\sqrt{18}a_1^1 a_2^{3^*} + \sqrt{27}a_1^1\, a_2^{1^*} + \sqrt{6}a_2^3\, a_2^{1^*}\right), \tag{30}$$

if $T = 1$ and $\pi = -1$, and

$$4\pi\, SpFF^+\, K_y^y = |a_1^2|^2 - |a_1^0|^2 + Re\left(\frac{\sqrt{2}}{2}a_1^2 a_1^{0^*} - \sqrt{\frac{15}{2}}a_2^2\, a_1^{0^*} + \sqrt{15}a_2^2\, a_1^{2^*}\right),$$
$$4\pi\, SpFF^+\, K_z^z = \frac{5}{2}|a_2^2|^2 - |a_1^0|^2 - \frac{1}{2}|a_1^2|^2 + Re\left(\sqrt{30}a_2^2\, a_1^{0^*} - \sqrt{2}a_1^2 a_1^{0^*} + \sqrt{15}a_2^2\, a_1^{2^*}\right), \tag{31}$$

if $T = 0$ $\pi = +1$. The factors $SpFF^+$ in Eqs. (30) and (31) are given by Eq. (20).

4.1.3 $j_4 = \frac{5}{2}$

We consider here the case of $j_4 = \frac{5}{2}$, because there is an experimental indication for this value of the spin of the $\Theta^+(1540)$ [10]. For $j_4 = \frac{5}{2}$ we find

$$4\pi\, SpFF^+\, K_y^y = 2(|a_2^3|^2 - |a_2^1|^2) + Re\left(2\sqrt{7}a_3^3 a_2^{3^*} - \frac{2\sqrt{42}}{3}a_3^3 a_2^{1^*} + \frac{\sqrt{6}}{3}a_2^3\, a_2^{1^*}\right),$$
$$4\pi\, SpFF^+\, K_z^z = \frac{7}{3}|a_3^3|^2 - |a_2^1|^2 - \frac{2}{3}|a_2^3|^2 + Re\left(\frac{8\sqrt{7}}{3}a_3^3 a_2^{3^*} + \frac{4\sqrt{42}}{3}a_3^3 a_2^{1^*} - \frac{2\sqrt{6}}{3}a_2^3\, a_2^{1^*}\right), \tag{32}$$

if $T = 1$ and $\pi = -1$, and

$$4\pi\, SpFF^{+}\, K^{y}_{y} = 4(|a^{2}_{3}|^{2} - |a^{4}_{3}|^{2}) - Re\left(\frac{2\sqrt{30}}{3} a^{2}_{2} a^{4^*}_{3} - \sqrt{10} a^{2}_{2}\, a^{2^*}_{3} + \frac{2\sqrt{3}}{3} a^{4}_{3}\, a^{2^*}_{3}\right),$$

$$4\pi\, SpFF^{+}\, K^{z}_{z} = -\frac{5}{3}|a^{2}_{2}|^{2} + \frac{4}{3}|a^{2}_{3}|^{2} + |a^{4}_{3}|^{2} + Re\left(\frac{4\sqrt{30}}{3} a^{2}_{2} a^{4^*}_{3} + \frac{8\sqrt{10}}{3} a^{2}_{2}\, a^{2^*}_{3} + \frac{4\sqrt{3}}{3} a^{4}_{3}\, a^{2^*}_{3}\right) \tag{33}$$

if $T = 0$ and $\pi = +1$.

4.2 Spin-spin correlation coefficients.

In this section we consider spin-spin correlations in the initial and final states of the reaction $1 + 2 \rightarrow 3 + 4$.

4.3 Spin-spin correlation in the initial state $\vec{j}_1 + \vec{j}_2 \rightarrow j_3 + j_4$

The initial spin-spin correlation coefficient is defined as [50]

$$C_{\lambda,\kappa} = \frac{SpF\, \hat{j}_{\lambda}(1)\, \hat{j}_{\kappa}(2)\, F^{+}}{SpF\, F^{+}\, j_1 j_2}, \tag{34}$$

where $\hat{j}_{\alpha}(i)$ $(\alpha = 0, \pm 1)$ is the spin operator of the i-th particle. Using Eqs.(1) and (34), we find for arbitrary spins j_i $(i = 1, \ldots, 4)$

$$\begin{aligned} C_{\lambda,\kappa}\, Sp\, FF^{+} = \delta_{\lambda,-\kappa} \frac{1}{4\pi} \sqrt{j_1^{-1}\, j_2^{-1}\, (j_1+1)(2j_1+1)(j_2+1)(2j_2+1)} \times \\ \times \sum_{S\, S'\, J} (-1)^{S+J} (2J+1) \sqrt{(2S+1)(2S'+1)} \times \\ \times \sum_{L\, L'\, J_0} (-1)^{L} \sqrt{(2J_0+1)(2L'+1)} \times \\ \times (1 \lambda\, 1 - \lambda\, |J_0\, 0)\, (J_0\, 0\, L'\, 0\, |L\, 0) \times \\ \times \begin{Bmatrix} S' & S & J_0 \\ L & L' & J \end{Bmatrix} \begin{Bmatrix} S' & j_1 & j_2 \\ S & j_1 & j_2 \\ J_0 & 1 & 1 \end{Bmatrix} a^{L\,S}_{J} (a^{L'\,S'}_{J'})^{*}. \end{aligned} \tag{35}$$

Using relations similar to those, which were employed for deriving of Eq.(24)(see Appendix), we find from Eq. (35) the following relations:

$$C_{+1,-1} = C_{-1,+1} = -C_{x,x} = -C_{y,y} \neq 0, \tag{36}$$

and $C_{0,0} = C_{z,z} \neq 0$, whereas

$$C_{i,j} = 0 \qquad \text{if } i \neq j,\ (i, j = x, y, z). \tag{37}$$

One can see from Eq. (34) that the following relation holds independently on the production mechanisms:

$$\Sigma = C_{x,x} + C_{y,y} + C_{z,z} = \hat{\mathbf{j}}(1) \cdot \hat{\mathbf{j}}(2)/(j_1\, j_2). \tag{38}$$

Therefore, Σ is fixed by the spin S. For $j_1 = j_2 = \frac{1}{2}$, one has $\Sigma = -3$ for $S = 0$ and $\Sigma = +1$ for $S = 1$ in accordance with the above results given in Eqs. (15), (16) and (8). From Eq. (35) one can find that $C_{x,x} = C_{y,y} = C_{z,z} = -1$ for $S = S' = 0$. For $S = S' = 1$, Eq. (35) is a generalization of Eqs.(15) and (16), derived for the particular case $j_1 = j_2 = \frac{1}{2}$

Using Eq.(35) one can write explicite formulae for $C_{i,j}$, which coincide with those obtained from the more simple formula in Eq.(15). We present below the formulae for $C_{y,y}$ for different j_4 at $j_1 = j_2 = j_3 = \frac{1}{2}$ and $S = 1$. The coefficient $C_{z,z}$ can be found as $C_{z,z} = 1 - 2C_{y,y}$.

4.3.1 $\quad j_4 = \frac{1}{2}$.

$$C_{y,y} = \frac{|a_0^1|^2}{|a_0^1|^2 + 3|a_1^1|^2}, \qquad \text{if } \pi = -1 \text{ and } T = 1; \tag{39}$$

$$C_{y,y} = \frac{|a_1^0 - \sqrt{2}a_1^2|^2}{3|a_1^0|^2 + 3|a_1^2|^2}, \qquad \text{if } \pi = +1 \text{ and } T = 0. \tag{40}$$

4.3.2 $j_4 = \frac{3}{2}$.

$$
\begin{aligned}
C_{y,y} &= \frac{|\sqrt{2}\, a_2^1 - \sqrt{3}\, a_2^3|^2}{3|a_1^1|^2 + 5|a_2^1|^2 + 5|a_2^3|^2}, && \text{if } \pi = -1 \text{ and } T = 1; \\
C_{y,y} &= \frac{|a_1^0 - \sqrt{2}\, a_1^2|^2}{3|a_1^0|^2 + 3|a_1^2|^2 + 5|a_2^2|^2}, && \text{if } \pi = +1 \text{ and } T = 0.
\end{aligned}
\tag{41}
$$

4.3.3 $j_4 = \frac{5}{2}$.

$$
\begin{aligned}
C_{y,y} &= \frac{|\sqrt{2}\, a_2^1 - \sqrt{3}\, a_2^3|^2}{5|a_2^1|^2 + 5|a_2^3|^2 + 7|a_3^3|^2}, && \text{if } \pi = -1 \text{ and } T = 1; \\
C_{y,y} &= \frac{|\sqrt{3} a_3^2 - 2\, a_3^4|^2}{5|a_2^2|^2 + 7|a_3^2|^2 + 7|a_3^4|^2}, && \text{if } \pi = +1 \text{ and } T = 0.
\end{aligned}
\tag{42}
$$

4.4 Spin-spin correlation in the final state $j_1 + j_2 \to \vec{j}_3 + \vec{j}_4$

By analogy with Eq. (34), for the spin-spin correlation in the final state, we can use the following definition

$$
C^f_{\lambda,\kappa} = \frac{Sp F^+\, \hat{j}_\lambda(3)\, \hat{j}_\kappa(4)\, F}{Sp F\, F^+\, j_3\, j_4}. \tag{43}
$$

We should note that in comparison with Eq. (34) the right hand side of Eq. (43) looks as the spin-spin correlation in the initial state of the inverse reaction $3 + 4 \to 1 + 2$. For arbitrary spins j_3 and j_4 we find from Eq. (43)

$$
\begin{aligned}
C^f_{\lambda,\kappa}\, Sp\, F F^+ = \delta_{\lambda,-\kappa}\, \frac{1}{4\pi} \sqrt{j_3^{-1}\, j_4^{-1}\, (j_3+1)(2j_3+1)\, (j_4+1)(2j_4+1)} \times \\
\times \sum_{J\, J'\, S} (-1)^{S+J} (2J+1)(2J'+1) \times \\
\times \sum_{L\, L'\, J_0} (-1)^L \sqrt{(2J_0+1)(2L'+1)} \times \\
\times (1 \lambda\, 1 - \lambda\, |\, J_0\, 0)\, (J_0\, 0\, L'\, 0\, |\, L\, 0) \times \\
\times \begin{Bmatrix} J' & S & L' \\ L & J_0 & J \end{Bmatrix} \begin{Bmatrix} J & j_3 & j_4 \\ J' & j_3 & j_4 \\ J_0 & 1 & 1 \end{Bmatrix} a_J^{L\, S} (a_{J'}^{L'\, S'})^*.
\end{aligned}
\tag{44}
$$

Here $J_0 = 0$ and 2. Others angular momenta J, J', S, L, L' are defined in the section 2.This formula is valid for arbitrary values of the spins j_i being integer or half-integer. We found from Eq. (44) the following relations: $C^f_{+1,-1} = C^f_{-1,+1} = -C^f_{x,x} = -C^f_{y,y} \neq 0$, $C^f_{0,0} = C^f_{z,z} \neq 0$, whereas $C^f_{i,j} = 0$ at $i \neq j$ $(i, j = x, y, z)$. Below we derive explicite formulae for $C^f_{\lambda,\kappa}$ for $j_4 = \frac{1}{2}$ and $\frac{3}{2}$, assuming $j_1 = j_2 = j_3 = \frac{1}{2}$.

One can see from Eq. (43) that the sum of the diagonal terms $C_{i,j}$ is determined by the total spin $\hat{\mathbf{J}} = \hat{\mathbf{j}}(3) + \hat{\mathbf{j}}(4)$ as

$$\Sigma^f = C^f_{x,x} + C^f_{y,y} + C^f_{z,z} =$$
$$= \hat{\mathbf{j}}(3) \cdot \hat{\mathbf{j}}(4)/(j_3 j_4) = \frac{1}{2 j_3 j_4} \left[J(J+1) - j_3(j_3+1) - j_4(j_4+1) \right]. \tag{45}$$

We will use below Eq.(45) as a check for the derived formulae for $C^f_{i,j}$.

4.4.1 *The case of $j_4 = \frac{1}{2}$*

One can find from Eq.(44)

$$C^f_{x,x} = C^f_{y,y} = C^f_{z,z} = -1, \text{ if } \pi = +1 \text{ and } T = 1 \tag{46}$$

and

$$C^f_{y,y} = -\frac{|a^1_0|^2}{|a^1_0|^2 + 3|a^1_1|^2}, C^f_{z,z} = \frac{3|a^1_1|^2 - |a^1_0|^2}{|a^1_0|^2 + 3|a^1_1|^2}, \text{ if } \pi = -1 \text{ and } T = 1$$
$$\Sigma^f = \frac{3|a^1_1|^2 - 3|a^1_0|^2}{|a^1_0|^2 + 3|a^1_1|^2} = \begin{cases} +1, \; J = 1, \\ -3, J = 0. \end{cases} \tag{47}$$

The final spin-spin correlation coefficients for $\pi = +1$ given by Eq. (46) differ from those for $\pi = -1$ given by Eq. (47). The only exception is the point where $a^1_1 = 0$ (but $a^1_0 \neq 0$).

For the case of $T = 0$ one has

$$C^f_{x,x} = C^f_{y,y} = +1, \; C^f_{z,z} = -1, \quad \text{if } \pi = -1 \text{ and } T = 0. \tag{48}$$

and

$$C^f_{y,y} = \frac{|a^0_1 - \sqrt{2}a^2_1|^2}{3(|a^0_1|^2 + |a^2_1|^2)}, \; C^f_{z,z} = 1 - 2C_{y,y}, \quad \text{if } \pi = +1 \text{ and } T = 0 \tag{49}$$

Eq. (49) coincides with Eq. (39). One can see from Eqs. (48) and (49), the final spin-spin correlation coefficients are different in absolute value for positive and negative parity π. Only exception is the point $a^2_1 = -\sqrt{2}a^0_1$ at which the coefficients $C^f_{j,j}$ for $\pi = +1$ are identical to those for $\pi = -1$ and have a maximal absolute value. One can see, Eq. (49) coincides with Eq. (40): $C^f_{x,x} = C^f_{y,y} = C^i_{x,x} = C^i_{y,y}$.

4.4.2 The case of $j_4 = \frac{3}{2}$

For $T = 1$ and $\pi = -1$ we find

$$C^f_{y,y} = \frac{1}{6}\frac{20(|a^1_2|^2 + |a^3_2|^2) - |3a^1_1 + \sqrt{3}a^1_2 + \sqrt{2}a^3_2|^2}{3|a^1_1|^2 + 5|a^1_2|^2 + 5|a^3_2|^2},$$
$$C^f_{z,z} = \frac{1}{3}\frac{|3a^1_1 + \sqrt{3}a^1_2 + \sqrt{2}a^3_2|^2 - 5(3|a^1_1 + |a^1_2|^2 + |a^3_2|^2)}{3|a^1_1|^2 + 5|a^1_2|^2 + 5|a^3_2|^2}. \tag{50}$$

$$\Sigma^f = \frac{1}{3}\frac{-15|a^1_1|^2 + 15|a^1_2|^2 + |a^3_2|^2}{3|a^1_1|^2 + 5|a^1_2|^2 + 5|a^3_2|^2} = \begin{cases} -\frac{5}{3}, & J = 1, \\ +1, & J = 2. \end{cases} \tag{51}$$

For $T = 1$ and $\pi = +1$ one has

$$C^f_{y,y} = +\frac{2}{3}, C^f_{z,z} = -\frac{1}{3}. \tag{52}$$

Let us consider the case of $T = 0$. For $\pi = +1$ we find

$$C^f_{y,y} = \frac{\frac{15}{6}|a^2_2|^2 - \frac{11}{6}|a^2_1|^2 - \frac{10}{6}|a^0_1|^2 - Re\left(\frac{\sqrt{15}}{3}\,a^2_2\,a^{2*}_1 + \frac{\sqrt{30}}{3}\,a^2_2 a^{0*}_1 - \frac{\sqrt{2}}{3}\,a^2_1 a^{0*}_1\right)}{3|a^0_1|^2 + 3|a^2_1|^2 + 5|a^2_2|^2},$$
$$C^f_{z,z} = \frac{-\frac{4}{3}|a^2_1|^2 - \frac{5}{3}|a^0_1|^2 + 2Re\left(\frac{\sqrt{15}}{3}\,a^2_2\,a^{2*}_1 + \frac{\sqrt{30}}{3}\,a^2_2 a^{0*}_1 - \frac{\sqrt{2}}{3}\,a^2_1 a^{0*}_1\right)}{3|a^0_1|^2 + 3|a^2_1|^2 + 5|a^2_2|^2},$$
$$\Sigma^f = \frac{5[|a^2_2|^2 - |a^2_1|^2 - |a^0_1|^2]}{3|a^0_1|^2 + 3|a^2_1|^2 + 5|a^2_2|^2} = \begin{cases} -\frac{5}{3}, & J = 1, \\ +1, & J = 2. \end{cases} \tag{53}$$

For $T=0$ and $\pi=-1$ one can find

$$C^f_{x,x}=C^f_{y,y}=-\frac{2}{3},C^f_{z,z}=-\frac{1}{3} \tag{54}$$

4.4.3 The case of $j_4=\frac{5}{2}$

For $T=1$ and $\pi=-1$

$$\begin{aligned}
C^f_{y,y}&=\frac{1}{15}\frac{56|a^3_3|^2-43|a^3_2|^2-42|a^1_2|^2-2Re\left(4\sqrt{7}a^3_3a^{3*}_2+2\sqrt{42}a^3_3a^{1*}_2-\sqrt{6}a^1_2a^{3*}_2\right)}{5|a^1_2|^2+5|a^3_2|^2+7|a^3_3|^2},\\
C^f_{z,z}&=\frac{1}{15}\frac{-7|a^3_3|^2-19|a^3_2|^2-21|a^1_2|^2+4Re\left[4\sqrt{7}a^3_3a^{3*}_2+2\sqrt{42}a^3_3a^{1*}_2-\sqrt{6}a^1_2a^{3*}_2\right]}{5|a^1_2|^2+5|a^3_2|^2+7|a^3_3|^2},\\
\Sigma^f&=7\frac{(|a^3_3|^2-|a^1_2|^2-|a^3_2|^2)}{5|a^1_2|^2+5|a^3_2|^2+7|a^3_3|^2}=\begin{cases}-\frac{7}{5}, & J=2,\\ +1, & J=3.\end{cases}
\end{aligned} \tag{55}$$

For the singlet initial state at $T=1$ and $\pi=+1$ we find

$$C^f_{x,x}=C^f_{y,y}=-\frac{3}{5},\ C^f_{z,z}=-\frac{1}{5}. \tag{56}$$

For $T=0$ and $\pi=+1$ we find

$$\begin{aligned}
C^f_{y,y}&=\frac{1}{15}\frac{-40|a^2_2|^2+60|a^4_3|^2+59|a^2_3|^2-4Re\left[\sqrt{30}a^2_2a^{4*}_3+2\sqrt{10}a^2_2a^{2*}_3+\sqrt{3}a^4_3a^{2*}_3\right]}{5|a^2_2|^2+7|a^4_3|^2+7|a^2_3|^2},\\
C^f_{z,z}&=\frac{1}{15}\frac{-25|a^2_2|^2-13|a^2_3|^2-15|a^4_3|^2+8Re\left[\sqrt{30}a^2_2a^{4*}_3+2\sqrt{10}a^2_2a^{2*}_3+\sqrt{3}a^4_3a^{2*}_3\right]}{5|a^2_2|^2+7|a^4_3|^2+7|a^2_3|^2},\\
\Sigma^f&=7\frac{(-|a^2_2|^2+|a^2_3|^2+|a^4_3|^2)}{5|a^2_2|^2+7|a^4_3|^2+7|a^2_3|^2}=\begin{cases}-\frac{7}{5}, & J=2,\\ +1, & J=3.\end{cases}
\end{aligned} \tag{57}$$

For the singlet initial state at $T=0$ and $\pi=-1$ one can find

$$C^f_{x,x}=C^f_{y,y}=\frac{3}{5},\ C^f_{z,z}=-\frac{1}{5}. \tag{58}$$

The spin-density matrix of the particle with the spin j is determined by $(2j+1)(2j+1)$ tensor momenta, t_{JM}. Here J is the rank ($J = 0, 1, \ldots, 2J$) and M is the magnetic quantum number ($M = -J, -J+1, \ldots, +J$). Let us define the spin-transfer coefficient for induced tensor polarization as

$$K_{J_1\,M_1}^{J_3\,M_3, J_4\,M_4} = \frac{Sp\,\{T_{J_3M_3}(3)T_{J_4M_4}(4)\,F\,T_{J_1M_1}(1)\,F^+\}}{SpFF^+}, \tag{59}$$

where $T_{J_iM_i}(i)$ is the tensor operator of the i-th particle, normalized as [51]

$$SpT_{JM}^+\,T_{J'M'} = \delta_{JJ'}\delta_{M'M}. \tag{60}$$

Using Eqs.(1), (18) and properties of the T_{JM} operators [51], we find the following formula

$$\begin{aligned}
K_{J_1M_1}^{J_3M_3,J_4M_4}\,SpFF^+ = \frac{1}{4\pi}\sqrt{(2J_1+1)(2J_3+1)(2J_4+1)}\\
\times \sum_{S\,S'\,J\,J'\,L\,L'\,J_0\,J_0'} (2J+1)(2J'+1)\sqrt{(2L+1)\,(2L'+1)}\;\times\\
\times\sqrt{(2S+1)\,(2S'+1)\,(2J_0+1)}(-1)^{(3(j_1+j_2)-S+L}\;\times\\
\times(J_0\,-M_1\,J_1\,M_1|J_0'\,0)(L'\,0\,L\,0\,|J_0'0)(J_3M_3J_4M_4|J_0-M_1)\;\times\\
\begin{Bmatrix} j_1 & j_2 & S' \\ S & J_1 & j_1 \end{Bmatrix}
\begin{Bmatrix} J' & j_3 & j_4 \\ J & j_3 & j_4 \\ J_0 & J_3 & J_4 \end{Bmatrix}
\begin{Bmatrix} J' & S' & L' \\ J & S & L \\ J_0 & J_1 & J_0' \end{Bmatrix}
a_J^{L\,S}(a_{J'}^{L'\,S'})^*.
\end{aligned} \tag{61}$$

Due to presence of the Clebsch-Gordan coefficient $(L'0L0|J_0'0)$ in Eq. (61) and P-parity conservation only even J_0' contribute to the right side of Eq.(61)

$$L + L' + J_0' \;\; is\;\; even, \;\;\; J_0' \;\; is\;\; even. \tag{62}$$

We are interesting here in the case of $M_4 = 0$ and J_4 being even, because for this case the tensor polarizations t_{J_40} can be measured via angular momentum distributions in the strong two-body decay of the Θ^+ (i.e. 4-th particle) into the nucleon and pseudoscalar meson [52]: $\Theta^+ \to N + K$. As it follows

from Eq.(61), the coefficient $K_{J_1 M_1}^{J_3 M_3, J_4 M_4}$ is non-zero only under the following condition between the magnetic quantum numbers

$$M_1 + M_3 + M_4 = 0. \tag{63}$$

If $J_1 + J_3 + J_4$ is *even*, the following relations are valid for $M_4 = 0$:

$$K_{J_1 +1}^{J_3 -1, J_4 0} = K_{J_1 -1}^{J_3 +1, J_4 0}, \;\; K_{J_1 +1}^{J_3 +1, J_4 0} = K_{J_1 -1}^{J_3 -1, J_4 0} = 0. \tag{64}$$

For $J_1 = J_3 = 1$ Eq. (64) can be rewritten in terms of Cartesian components as

$$K_{1\,x}^{1\,x, J_4 0} = K_{1\,y}^{1\,y, J_4 0} = -K_{1\,-1}^{1\,+1, J_4 0} = -K_{1\,+1}^{1\,-1, J_4 0},$$
$$K_{1\,\alpha}^{1\,\beta, J_4 0} = 0 \;\text{ if } \alpha \neq \beta \;\; (\alpha, \beta = x, y, z). \tag{65}$$

The coefficient $K_{J_1\,0}^{J_3\,0, J_4\,0}$ can be non-zero only for $J_1 + J_3 + J_4$ being even,

$$K_{J_1 0}^{J_3 0, J_4 0} \neq 0, \;\text{ if } J_1 + J_3 + J_4 \text{ is even.} \tag{66}$$

If $J_1 + J_3 + J_4$ is *odd*, then one can find

$$K_{J_1 +1}^{J_3 -1, J_4 0} = -K_{J_1 -1}^{J_3 +1, J_4 0}; \;\; K_{1\,+1}^{1\,-1, J_4 0} = iK_{1\,x}^{1\,y, J_4 0} = -iK_{1\,y}^{1\,x, J_4 0}, \tag{67}$$
$$K_{1\,x}^{1\,x, J_4 0} = K_{1\,y}^{1\,y, J_4 0} = 0, \tag{68}$$
$$K_{J_1\,0}^{J_3\,0, J_4\,0} = 0. \tag{69}$$

Eq.(61) is rather general and can be used for calculation of the above considered spin observables. So, if one put in Eq.(61) $J_4 = 0$ and $M_4 = 0$, then the spin transfer coefficient K_α^β (α, $\beta = x, y, z$), given by Eq.(23), can be obtained from Eq.(61) as

$$K_\alpha^\beta = \frac{1}{3}\sqrt{j_1^{-1}\, j_3^{-1}(j_1+1)(2j_1+1)(j_3+1)(2j_3+1)(2j_4+1)}\,K_{1\,\alpha}^{1\,\beta, 00}. \tag{70}$$

When substituting $J_1 = 0$ and $M_1 = 0$ into Eq.(61), one can find spin-tensor correlation in the final state. In particular, Eq. (44) follows from Eq.(61) at $J_1 = 0,\ M_1 = 0,\ J_4 = 1$.

For the spin-singlet initial state $S = S' = 0$ the coefficients $K_{J_1M_1}^{J_3M_3,J_4M_4}$ are non-zero only for unpolarized beam (or target), i.e. $J_1 = M_1 = 0$, as it follows from presence of the second 9j-symbol in Eq. (61)

$$K_{K_1M_1}^{J_3M_3,J_4M_4} = \delta_{J_1,0}\,\delta_{M_1,0}\,K_{00}^{J_3M_3,J_4M_4},\ \text{if}\ S = S' = 0. \tag{71}$$

For $J_1 = M_1 = 0$ one has $J_0 = J_0'$, where J_0' is even, according to Eq. (62). Furthermore, at $j_3 = \frac{1}{2}$ two values are allowed for the rank J_3: $J_3 = 0$ and $J_3 = 1$. (i) If $J_3 = M_3 = 0$, then only even ranks J_4 are allowed, as follows from the Clebsch-Gordan coefficient $(J_3\,M_3\,J_4\,M_4|J_0\,-M_1)$ in Eq.(61). (ii) For $J_3 = 1$ the non-zero coefficients $K_{00}^{1M_3,J_4M_4}$ are allowed for $M_3 = -M_4 \neq 0$ both for even and odd J_4. However, for the practically interesting case of $M_4 = 0$ one has $M_3 = 0$ due to Eq.(63). Therefore for $J_1 = 0$, $M_4 = 0$ and *even* J_4 the value $J_3 = 1$ is not allowed due to Eq.(66). Thus, for unpolarized beam and $M_4 = 0$ only the coefficients K_{00}^{00,J_40} with even rank J_4 are non-zero:

$$K_{00}^{00,J_40} \neq 0,\ \text{if}\ J_4\ \text{is even}. \tag{72}$$

This result is valid both for the spin-singlet and spin-triplet initial states.

4.5.1 Tensor polarization for $j_1 + j_2 \to j_3 + \vec{j}_4$

Here we consider the tensor polarization of the *4-th* particle, t_{J_40}, when all others particles are unpolarized in the binary reaction $1 + 2 \to 3 + \vec{4}$. The tensor polarization of the *4-th* particle can be calculated as

$$t_{J_4M_4} = \frac{SpFF^+T_{J_4M_4}(4)}{SpFF^+}. \tag{73}$$

Using Eq.(59) and the normalization of T_{JM} operator given by Eq.(60), one can find the relation

$$t_{J_40} = \sqrt{(2j_1+1)(2j_3+1)}K_{00}^{00,J_40}. \tag{74}$$

For the *spin-singlet state* $S = S' = 0$, the tensor polarization of the *4-th*-particle does not depend on the dynamics of the reaction $1+2 \to 3+4$ at the threshold and intrinsic parity of particles and takes the following values

$$t_{00} = \frac{1}{2},\ t_{20} = -\frac{1}{2}, \quad \text{for } j_4 = \frac{3}{2} \text{ and } \pi = \pm 1, \tag{75}$$

$$t_{00} = \frac{1}{\sqrt{6}},\ t_{20} = -\frac{2}{\sqrt{21}}, t_{40} = \frac{1}{\sqrt{7}}, \quad \text{for } j_4 = \frac{5}{2} \text{ and } \pi = \pm 1, \tag{76}$$

$$\tag{77}$$

For the *spin-triplet state* $S = 1$ we find the following formulae for $t_{J_4 0}$ with the even rank J_4:

$$4\pi SpFF^{+}\, t_{20} = \frac{3}{4}|a_1^1|^2 - 2|a_2^3|^2 - \frac{7}{4}|a_2^1|^2 - \tag{78}$$
$$-Re\left(\frac{3}{\sqrt{2}}a_1^1 a_2^{3*} + \frac{3\sqrt{3}}{2}a_1^1 a_2^{1*} - \frac{\sqrt{6}}{2}a_2^3\, a_2^{1*}\right),$$

for $j_4 = \frac{3}{2}$, $T = 1$, $\pi = -1$,

$$4\pi SpFF^{+}\, t_{20} = -\frac{5}{4}|a_2^2|^2 - \frac{3}{4}|a_1^2|^2 - \tag{79}$$
$$-\frac{1}{2}Re\left(\sqrt{15}a_2^2 a_1^{2*} + \sqrt{30}a_2^2 a_1^{0*} - 3\sqrt{2}a_1^2\, a_1^{0*}\right),$$

for $j_4 = \frac{3}{2}$, $T = 0$, $\pi = +1$,

$$4\pi SpFF^{+}\, t_{20} = -\frac{\sqrt{21}}{3}|a_2^2|^2 - \frac{8}{\sqrt{21}}|a_2^4|^2 - \frac{\sqrt{21}}{2}|a_3^3|^2 - \tag{80}$$
$$-Re\left(\frac{2}{3}a_3^3 a_2^{4*} + \sqrt{2}a_3^3 a_2^{2*} - 2\frac{\sqrt{14}}{7}a_2^4\, a_2^{2*}\right),$$

for $j_4 = \frac{5}{2}$, $T = 1$, $\pi = -1$,

$$4\pi SpFF^{+}\, t_{20} = -\frac{5}{\sqrt{21}}|a_2^2|^2 - \frac{25}{2\sqrt{21}}|a_3^4|^2 - \frac{12}{\sqrt{21}}|a_3^2|^2 - \tag{81}$$
$$-Re\left(\sqrt{\frac{10}{7}}a_2^2 a_3^{4*} + 2\sqrt{\frac{10}{21}}a_2^2 a_3^{2*} - \frac{2}{\sqrt{7}}a_3^4\, a_3^{2*}\right),$$

for $j_4 = \frac{5}{2}$, $T = 0$, $\pi = +1$,

$$4\pi SpFF^{+}\, t_{40} = \frac{\sqrt{7}}{6}|a_3^3|^2 + \frac{5\sqrt{7}}{21}|a_2^3|^2 + \tag{82}$$
$$+Re\left(\sqrt{\frac{10}{3}}a_3^3 a_2^{1*} + \frac{10}{3}a_3^3 a_2^{3*} - 10\sqrt{\frac{2}{21}}a_2^3\, a_2^{1*}\right),$$

for $j_4 = \frac{5}{2}$, $T = 1$, $\pi = -1$, and

$$4\pi SpFF^{+}\, t_{40} = \frac{\sqrt{7}}{42}\big\{-20|a_2^2|^2 + 27|a_3^4|^2 + 22|a_3^2|^2 +$$
$$+10Re\left(\sqrt{30}\, a_2^2 a_3^{4*} + 2\sqrt{10}a_2^2 a_2^{3*} - 2\sqrt{3}a_3^4\, a_3^{2*}\right)\big\}, \tag{83}$$

for $j_4 = \frac{5}{2}$, $T = 0$, $\pi = +1$. The formulae for $SpFF^{+}$ are given by Eq. (20).

The tensor polarization t_{J0} of the odd rank J vanishes, as can be seen from Eq. (72).

4.5.2 Induced tensor polarization for $\vec{j}_1 + j_2 \to j_3 + \vec{j}_4$

As follows from Eq. (71), for the *spin-singlet state* the tensor polarization of the *4-th* particle cannot be induced by polarized beam ($J_1 \neq 0$).

For *the spin-triplet* state $S = S' = 1$ and $J_1 \neq 0$ the non-zero coefficient $K^{00,J_4 0}_{J_1 M_1} \neq 0$ is allowed only for $M_1 = 0$ as it follows from Eq.(63), i.e. for longitudinally polarized beam. Furthermore, according to Eq. (66), only odd rank J_4 is allowed for $K^{00,J_4\, 0}_{10}$. Since the case with odd J_4 is not interesting for our purpose, we do not consider here this option in detail.

4.6 Spin-tensor correlation for $j_1 + j_2 \to \vec{j}_3 + \vec{j}_4$

Assuming the spin of the *3*-rd particle is $j_3 = \frac{1}{2}$, we find for $M_4 = 0$ that the spin-tensor correlation in the final state of the reaction $j_1 + j_2 \to \vec{j}_3 + \vec{j}_4$ for unpolarized beam ($J_1 = M_1 = 0$) and target ($J_2 = M_2 = 0$) is non-zero ($K^{1\,0,\,J_4\,0}_{00} \neq 0$) only for odd rank J_4 (see Eq.(66)).

For $j_3 \geq \frac{3}{2}$ and $j_4 \geq \frac{3}{2}$ the spin-tensor correlation coefficient $K_{00}^{J_3\,0,J_4\,0}$ could be non-zero for J_3 and J_4 being even, however, the number of independent spin amplitudes increases in this case that makes the relation between this coefficient and P-parity non-transparent.

4.6.1 The observables for $\vec{j}_1 + j_2 \to \vec{j}_3 + \vec{j}_4$

We consider below the final spin-tensor correlation, $K_{1y}^{1y,J0}$, induced by the transversally and longitudinally polarized beam (or target) in the reaction $\vec{\frac{1}{2}} + \frac{1}{2} \to \vec{\frac{1}{2}} + \vec{j}_4$. This coefficient is non-zero only for the spin-triplet initial state.

For the even rank J one can find from Eq. (61)

$$4\pi SpFF^+\,K_{1\,y}^{1\,y,20} = \frac{3}{4}(|a_2^3|^2 - |a_2^1|^2) +$$
$$+Re\Big(\frac{3\sqrt{2}}{8}a_1^1a_2^{3^*} + \frac{\sqrt{6}}{8}a_2^3a_2^{1^*} - \frac{\sqrt{3}}{4}a_1^1a_2^{1^*}\Big), \qquad (84)$$
$$4\pi SpFF^+\,K_{1\,z}^{1\,z,20} = -\frac{1}{4}\Big\{3|a_1^1|^2 + 2|a_2^3|^2 + 3|a_2^1|^2 + 2\sqrt{6}Re\,a_2^3\,a_2^{1^*}\Big\}, \qquad (85)$$

for $j_4 = \frac{3}{2}$, $T = 1$, $\pi = -1$,

$$4\pi SpFF^+\,K_{1\,y}^{1\,y,20} = \frac{1}{4}\Big\{|a_1^0|^2 - |a_1^2|^2 +$$
$$+Re\Big(\frac{\sqrt{30}}{2}a_2^2a_1^{0^*} - \frac{\sqrt{2}}{2}a_1^2a_1^{0^*} - \sqrt{15}a_2^2a_1^{2^*}\Big)\Big\}, \qquad (86)$$
$$4\pi SpFF^+\,K_{1\,z}^{1\,z,20} = -\frac{1}{4}\Big\{2|a_1^0|^2 + |a_1^2|^2 + 5|a_2^2|^2 + 2\sqrt{2}Re\,a_1^2a_1^{0^*}\Big\} \qquad (87)$$

for $j_4 = \frac{3}{2}$, $T = 0$, $\pi = +1$,

$$4\pi SpFF^+\,K_{1\,y}^{1\,y,20} = \frac{2}{\sqrt{21}}\Big(|a_2^1|^2 - |a_2^3|^2\Big) +$$
$$+Re\Big(\frac{2\sqrt{2}}{3}a_3^3a_2^{1^*} - \frac{2\sqrt{3}}{3}\,a_3^3a_2^{3^*} - \frac{\sqrt{14}}{21}\,a_2^1a_2^{3^*}\Big), \qquad (88)$$
$$4\pi SpFF^+\,K_{1\,z}^{1\,z,20} = -\frac{1}{252}\Big\{\sqrt{21}\Big(49|a_3^3|^2 + 4|a_2^3|^2 + 6|a_2^1|^2\Big) +$$
$$+Re\Big(210\sqrt{2}\,a_3^3a_2^{1^*} + 12\sqrt{14}\,a_2^3a_2^{1^*} + 140\sqrt{3}\,a_3^3a_2^{3^*}\Big)\Big\}, \qquad (89)$$

for $j_4 = \frac{5}{2}$, $T = 1$, $\pi = -1$,

$$4\pi SpFF^+ \, K_{1\,y}^{1\,y,20} = \frac{2\sqrt{21}}{13}\Big(|a_3^4|^2 - |a_3^2|^2\Big) -$$
$$-\frac{\sqrt{7}}{42} Re\Big(4\sqrt{10} a_2^2 a_3^{4*} - 2\sqrt{30} a_2^2 a_3^{2*} + 4 a_3^4 a_3^{2*}\Big), \qquad (90)$$
$$4\pi SpFF^+ \, K_{1\,z}^{1\,z,20} = -\frac{\sqrt{7}}{252}\Big\{\sqrt{3}\Big(21|a_3^4|^2 + 28|a_3^2|^2 + 10|a_2^2|^2\Big) +$$
$$+Re\Big(30\sqrt{10}\, a_2^2 a_3^{4*} + 20\sqrt{30}\, a_2^2 a_3^{2*} + 84\, a_3^4 a_3^{2*}\Big)\Big\}, \qquad (91)$$

for $j_4 = \frac{5}{2}$, $T = 0$, $\pi = +1$,

$$4\pi SpFF^+ \, K_{1\,y}^{1\,y,40} = \frac{\sqrt{7}}{7}\Big(|a_2^3|^2 - |a_2^1|^2\Big) +$$
$$+Re\Big(a_3^3 a_2^{3*} - \frac{\sqrt{6}}{3} a_3^3 a_2^{1*} + \frac{1}{\sqrt{42}} a_2^3 a_2^{1*}\Big)\Big\}, \qquad (92)$$
$$4\pi SpFF^+ \, K_{1\,z}^{1\,z,40} = \frac{1}{84}\Big\{\sqrt{7}\Big(16|a_2^3|^2 + 24|a_2^1|^2 + 49|a_3^3|^2\Big) +$$
$$+Re\Big(-28 a_3^3 a_2^{3*} - 14\sqrt{6} a_3^3 a_2^{1*} + 16\sqrt{42} a_2^3 a_2^{1*}\Big)\Big\} \qquad (93)$$

for $j_4 = \frac{5}{2}$, $T = 1$, $\pi = -1$, and

$$4\pi SpFF^+ \, K_{1\,y}^{1\,y,40} = \frac{2}{\sqrt{7}}\Big(|a_3^2|^2 - |a_3^4|^2\Big) +$$
$$+Re\Big(-\frac{\sqrt{30}}{21} a_2^2 a_3^{4*} + \frac{\sqrt{10}}{14} a_2^2 a_3^{2*} - \frac{\sqrt{3}}{21} a_3^4 a_3^{2*}\Big)\Big\}, \qquad (94)$$
$$4\pi SpFF^+ \, K_{1\,z}^{1\,z,40} = \frac{\sqrt{7}}{84}\Big\{28|a_3^2|^2 + 21|a_3^4|^2 + 40|a_2^2|^2 -$$
$$-Re\Big(2\sqrt{30}\, a_2^2 a_3^{4*} + 4\sqrt{10}\, a_2^2 a_3^{2*} - 28\,\sqrt{3} a_3^4 a_3^{2*}\Big)\Big\}, \qquad (95)$$

for $j_4 = \frac{5}{2}$, $T = 0$, $\pi = +1$.

For the odd rank J, one has $K_{00}^{00,J0} = 0$, therefore the tensor polarization of the *4th* particle $t_{10}, t_{30}, \ldots,$ is equal to zero if the beam is unpolarized and the polarization of the *3-rd* particle is not measured. However, the tensor polarization of the odd rank can be induced by polarized beam in the transitions from the spin-triplet initial state. So, for $S = 1$ we obtained the following results:

$$K_{1\,x}^{1\,y,10} = -\frac{1}{2}\frac{Im\, a_1^2 a_1^{0*}}{|a_1^2|^2 + |a_1^0|^2}, \tag{96}$$

for $j_4 = \frac{1}{2}$, $T = 0$, $\pi = +1$;

$$K_{1\,x}^{1\,y,10} = -\frac{\sqrt{3}}{2}\frac{Im\, a_1^1 a_0^{1*}}{3|a_1^1|^2 + |a_0^1|^2}, \tag{97}$$

for $j_4 = \frac{1}{2}$, $T = 1$, $\pi = -1$;

$$4\pi SpFF^+\, K_{1\,x}^{1\,y,10} = -\frac{1}{40} Im\Big(10\sqrt{3} a_2^2 a_1^{2*} + 5\sqrt{6}\, a_1^0 a_2^{2*} + 3\sqrt{10} a_1^0 a_1^{2*}\Big), \tag{98}$$

for $j_4 = \frac{3}{2}$, $T = 0$, $\pi = +1$;

$$4\pi SpFF^+\, K_{1\,x}^{1\,y,10} = \frac{\sqrt{5}}{40} Im\Big(3\sqrt{2}\, a_1^1 a_2^{3*} + 2\sqrt{3}\, a_2^1 a_1^{1*} + 5\sqrt{6} a_2^1 a_2^{3*}\Big), \tag{99}$$

for $j_4 = \frac{3}{2}$, $T = 1$, $\pi = -1$;

$$\begin{aligned} 4\pi SpFF^+\, K_{1\,x}^{1\,y,30} &= \frac{\sqrt{5}}{45} Im\Big(6\sqrt{7} a_3^3 a_2^{3*} + 2\sqrt{42}\, a_2^1 a_3^{3*} + 5\sqrt{6} a_2^1 a_2^{3*}\Big), \\ 4\pi SpFF^+\, K_{1\,x}^{1\,y,10} &= -\frac{\sqrt{105}}{210} Im\Big(\sqrt{42}\, a_3^3 a_2^{3*} + \frac{14}{\sqrt{7}}\, a_2^1 a_3^{3*} + 5 a_2^1 a_2^{3*}\Big), \end{aligned} \tag{100}$$

for $j_4 = \frac{5}{2}$, $T = 1$, $\pi = -1$;

$$\begin{aligned} 4\pi SpFF^+\, K_{1\,x}^{1\,y,30} &= -\frac{1}{3} Im\Big(\frac{2\sqrt{6}}{3}\, a_2^2 a_3^{4*} + \sqrt{2}\, a_3^2 a_2^{2*} + \frac{14}{\sqrt{15}} a_3^2 a_3^{4*}\Big), \\ 4\pi SpFF^+\, K_{1\,x}^{1\,y,10} &= \frac{\sqrt{7}}{210} Im\Big(10\sqrt{3}\, a_2^2 a_3^{4*} + 15\, a_3^2 a_2^{2*} + 7\sqrt{30} a_3^2 a_3^{4*}\Big), \end{aligned} \tag{101}$$

for $j_4 = \frac{5}{2}$, $T = 0$, $\pi = +1$.

5 Full spin structure for the reaction $\frac{1}{2} + \frac{1}{2} \to \frac{1}{2} + \frac{1}{2}$

Here we give the full spin structure of the binary reaction for $j_1 = j_2 = j_3 = j_4 = \frac{1}{2}$.

5.1 *The reaction $pn \to \Lambda^0\Theta^+$*

5.1.1 *The negative parity*

For the case $T=0$ and $\pi=-1$, one has $S=0$. In this case the amplitude (1) describes the transition ${}^1P_1 \to {}^3S_1$ and can be written as

$$M^{\mu_3\,\mu_4}_{\mu_1\,\mu_2} = \sum_{\alpha=x,y,z} (\chi^+_{\mu_3}\sigma_\alpha \frac{i\sigma_y}{\sqrt{2}}\,\chi^{(T)+}_{\mu_4})\,(\chi^{(T)}_{\mu_1}\frac{-i\sigma_y}{\sqrt{2}}\,\chi_{\mu_2})\hat{k}_\alpha\sqrt{\frac{3}{4\,\pi}}\,a^{1\,0}_1. \tag{102}$$

When deriving Eq. (102) from Eq. (1) we used for the Clebsch-Gordan coefficients the formulae given above after Eq. (4). The unpolarized cross section corresponding to the amplitude (102) takes the following form:

$$d\sigma_0 = \frac{1}{4}\Phi \sum_{\mu_1\,\mu_2\,\mu_3\,\mu_4} |M^{\mu_3\,\mu_4}_{\mu_1\,\mu_2}|^2 = \frac{3}{16\pi}\Phi\,|a^{1\,0}_1|^2 \tag{103}$$

that is in agreement with Eq. (7). In order to calculate the polarized cross section we use the density matrix for the spin-$\frac{1}{2}$ particle being in the pure spin state χ_{μ_i} in the following form:

$$\chi_{\mu_i}\,\chi^+_{\mu_i} = \frac{1}{2}(1+\boldsymbol{\sigma}\cdot\mathbf{p}_i). \tag{104}$$

Using Eqs.(104) and (102) one can write the cross section with polarized both initial and final particles as

$$\begin{gathered} d\sigma(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p}_3,\mathbf{p}_4) = \Phi\,|M^{\mu_3\,\mu_4}_{\mu_1\,\mu_2}|^2 = \\ = \frac{1}{4}d\sigma_0\,(1-\mathbf{p}_1\cdot\mathbf{p}_2)\,[1+\mathbf{p}_3\cdot\mathbf{p}_4 - 2(\mathbf{p}_3\cdot\hat{\mathbf{k}})(\mathbf{p}_4\cdot\hat{\mathbf{k}})]. \end{gathered} \tag{105}$$

The polarization vectors of the final particles $\mathbf{p}_3$ and $\mathbf{p}_4$ are determined by the reaction amplitude (102) and can be found using the standard methods [46,50]. After performing this step and substituting the obtained vectors $\mathbf{p}_3$ and $\mathbf{p}_4$ into Eq. (105), one can find the polarized cross section $d\sigma(\mathbf{p}_1,\mathbf{p}_2)$ given by Eq. (8). However, the calculation of $\mathbf{p}_3$ and $\mathbf{p}_4$ is not necessarily and Eq. (105) is sufficient to find all spin observables for the reaction described by the amplitude (102). In particular, one can see from Eq. (105) that there is

no polarization transfer in this reaction ($K_i^j = 0$, $i, j = x, y, z$), but there are spin-spin correlations in both the initial and final states.

5.1.2 *The positive parity*

For $T = 0$ and $\pi = +1$ we have $S = 1$. In this case the amplitude in Eq.(1) describes the transition ${}^3S_1 - {}^3D_1 \to {}^3S_1$ and can be written as

$$M^{\mu_3\,\mu_4}_{\mu_1\,\mu_2} = \sum_{\alpha=x,y,z} (\chi^+_{\mu_3}\sigma_\alpha \frac{i\sigma_y}{\sqrt{2}}\,\chi^{(T)+}_{\mu_4})\,(\chi^{(T)}_{\mu_1}\frac{-i\sigma_y}{\sqrt{2}}\,\Pi_\alpha\chi_{\mu_2}), \tag{106}$$

where Π_α is the following spin operator

$$\Pi_\alpha = G\sigma_\alpha + F\,\hat{k}_\alpha(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}) \tag{107}$$

with

$$G = \frac{1}{\sqrt{4\pi}}\,(a^0_1 + \frac{1}{\sqrt{2}}\,a^2_1) \tag{108}$$

and

$$F = -\frac{3}{\sqrt{8\pi}}\,a^2_1. \tag{109}$$

The cross section with polarized initial and final particles is the following

$$\begin{aligned} d\sigma(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}_3, \mathbf{p}_4) = \Phi\,|M^{\mu_3\,\mu_4}_{\mu_1\,\mu_2}|^2 = \\ \times \sum_{\alpha\,\beta=x,y,z} \frac{1}{8} Sp\{\sigma_\alpha\,(1-\boldsymbol{\sigma}\cdot\mathbf{p}_4)\,\sigma_\beta(1+\boldsymbol{\sigma}\cdot\mathbf{p}_3)\} \times \\ \times\frac{1}{8} Sp\{\Pi^+_\alpha\,(1+\boldsymbol{\sigma}\cdot\mathbf{p}_2)\,\Pi_\beta(1-\boldsymbol{\sigma}\cdot\mathbf{p}_1)\}. \end{aligned} \tag{110}$$

Calculating the traces in Eq. (110), one can find finally

$$
\begin{aligned}
d\sigma(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p}_3,\mathbf{p}_4) = \frac{1}{4}d\sigma_0\Big\{1 + \Big(|F+G|^2+2|G|^2\Big)^{-1}\Big\{|F+G|^2(\mathbf{p}_1\cdot\mathbf{p}_2+\mathbf{p}_3\cdot\mathbf{p}_4) - \\
+2\Big(|F|^2+2Re\,FG^*\Big)\Big[(\mathbf{p}_1\cdot\hat{\mathbf{k}})(\mathbf{p}_2\cdot\hat{\mathbf{k}})+(\mathbf{p}_3\cdot\hat{\mathbf{k}})(\mathbf{p}_4\cdot\hat{\mathbf{k}})\Big] + \\
+2\Big(|G|^2+2Re\,FG^*\Big)(\mathbf{p}_1+\mathbf{p}_2)\cdot(\mathbf{p}_3+\mathbf{p}_4) - 2Re\,FG^*(\mathbf{p}_1\cdot\hat{\mathbf{k}}+\mathbf{p}_2\cdot\hat{\mathbf{k}})(\mathbf{p}_3\cdot\hat{\mathbf{k}}+\mathbf{p}_4\cdot\hat{\mathbf{k}}) + \\
+2Im\,FG^*\Big[\Big\{\Big((\mathbf{p}_3\cdot\hat{\mathbf{k}})\,[\mathbf{p}_4\times\hat{\mathbf{k}}]+(\mathbf{p}_4\cdot\hat{\mathbf{k}})[\mathbf{p}_3\times\hat{\mathbf{k}}]\Big)\cdot(\mathbf{p}_1+\mathbf{p}_2)\Big\} - \Big\{\ldots 1\leftrightarrow 3, 2\leftrightarrow 4\ldots\Big\}\Big] + \\
+4|F|^2(\mathbf{p}_1\cdot\hat{\mathbf{k}})(\mathbf{p}_2\cdot\hat{\mathbf{k}})(\mathbf{p}_3\cdot\hat{\mathbf{k}})(\mathbf{p}_4\cdot\hat{\mathbf{k}}) - \\
-2\Big(|F|^2+2Re\,FG^*\Big)\Big[(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{p}_3\cdot\hat{\mathbf{k}})(\mathbf{p}_4\cdot\hat{\mathbf{k}})+(\mathbf{p}_1\cdot\hat{\mathbf{k}})(\mathbf{p}_2\cdot\hat{\mathbf{k}})(\mathbf{p}_3\cdot\mathbf{p}_4)\Big] + \\
+2\,Re\,FG^*\Big[\Big\{(\mathbf{p}_1\cdot\hat{\mathbf{k}})(\mathbf{p}_3\cdot\hat{\mathbf{k}})(\mathbf{p}_4\cdot\mathbf{p}_2)+(\mathbf{p}_1\cdot\hat{\mathbf{k}})(\mathbf{p}_4\cdot\hat{\mathbf{k}})(\mathbf{p}_2\cdot\mathbf{p}_3)\Big\}+\Big\{\ldots 1\leftrightarrow 2\ldots\Big\}\Big] + \\
+\Big(|F+G|^2-2|G|^2\Big)(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{p}_3\cdot\mathbf{p}_4)+2|G|^2\Big[(\mathbf{p}_1\cdot\mathbf{p}_3)(\mathbf{p}_2\cdot\mathbf{p}_4)+(\mathbf{p}_2\cdot\mathbf{p}_3)(\mathbf{p}_1\cdot\mathbf{p}_4)\Big]\Big\}\Big\}.
\end{aligned}
\tag{111}
$$

The unpolarized cross section for this case is the following

$$d\sigma_0 = \frac{1}{4}\Phi\,\Big\{|G+F|^2+2|G|^2\Big\} = \frac{\Phi}{16\pi}3(|a_1^0|^2+|a_1^2|^2), \tag{112}$$

were we used Eqs. (108) and (109). All spin observables for this reaction are contained in Eq. (111). For example, the spin-spin correlation coefficient $C_{i,j}$ is a factor in front of the product of the polarization vectors of the *1*-st and *2*-nd particles, $p_{1\,i}\,p_{2\,j}$, in Eq.(111). One can find these coefficients as

$$C_{x,x} = \frac{|G+F|^2}{|G+F|^2+2|G|^2},\ \ C_{z,z} = \frac{|G-F|^2-2|F|^2}{|G+F|^2+2|G|^2}, \tag{113}$$

which coincide with Eqs. (15) and (16), respectively. The spin transfer coefficient K_i^j is the factor in front of the product of the polarization vectors of the *1*-st and *3*-rd particles, $p_{1\,i}\,p_{3\,j}$, in Eq.(111). One can find from Eq.(111)

$$K_x^x = 2\frac{|G|^2+Re\,FG^*}{|G+F|^2+2|G|^2},\ \ K_z^z = \frac{2|G|^2}{|G+F|^2+2|G|^2}, \tag{114}$$

which coincide with Eqs. (26) and (27), respectively. One can find also from Eq. (111) the coefficient $K_{1x}^{1y,\,1z}$ as [6]

[6] Here $K_{1x}^{1y,10}$ is defined by Eq. (59) and the additional factor $2\sqrt{2}$ in Eq.(115) follows from the relation between the spin-tensor T_{1M} and σ-matrix: $\sigma_M = \sqrt{2}T_{1M}$.

$$
\begin{aligned}
&K_{1x}^{1y,\,1z} = K_{1x}^{1z,\,1y} = -K_{1y}^{1z,\,1x} = K_{1y}^{1x,\,1z} = \\
&= \frac{1}{2\sqrt{2}}\frac{2Im\,FG^*}{|G+F|^2+2|G|^2} = -\frac{1}{2}\frac{Im\,a_1^2 a_1^{0*}}{|a_1^0|^2+|a_1^2|^2},
\end{aligned} \tag{115}
$$

that coincides with Eq. (96). One can find from Eq. (111) also the coefficients in front of the products $p_{1\,i}\,p_{2\,j}\,p_{3\,k}$, which we denote here as $\bar{K}_{i,j}^{k}$. These coefficients describe the spin-spin correlation in the initial state, which induces the polarization of the final particle:

$$
\bar{K}_{z,y}^{x} = -\bar{K}_{z,x}^{y} = \bar{K}_{y,z}^{x} = -\bar{K}_{x,z}^{y} = \frac{2Im\,FG^*}{|G+F|^2+2|G|^2}. \tag{116}
$$

For the spin-singlet initial state (see Eq.(105)) these coefficients equal zero.

5.2 *The reaction $pp \to \Sigma^+\Theta^+$*

In the reaction $pp \to \Sigma^+\Theta^+$ the initial state is an isotriplet, $T = 1$. Therefore, for $\pi = +1$ one has $S = 0$ whereas for $\pi = -1$ one obtains $S = 1$.

5.2.1 *The positive parity*

In case of $\pi = +1$ the amplitude (1) describes the transition ${}^1S_0 \to {}^1S_0$ and can be written as

$$
M_{\mu_1\,\mu_2}^{\mu_3\,\mu_4} = \sqrt{\frac{1}{4\,\pi}}\,(\chi_{\mu_3}^{+}\,\frac{i\sigma_y}{\sqrt{2}}\,\chi_{\mu_4}^{(T)+})\,(\chi_{\mu_1}^{(T)}\frac{-i\sigma_y}{\sqrt{2}}\,\chi_{\mu_2})\hat{k}_\alpha\,a_0^{0\,0}. \tag{117}
$$

The cross section with polarized initial and final particles is the following

$$
\begin{aligned}
&d\sigma(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p}_3,\mathbf{p}_4) = \Phi\,|M_{\mu_1\,\mu_2}^{\mu_3\,\mu_4}|^2 = \\
&= \frac{1}{4}d\sigma_0\,(1-\mathbf{p}_1\cdot\mathbf{p}_2)\,(1-\mathbf{p}_3\cdot\mathbf{p}_4).
\end{aligned} \tag{118}
$$

One can see from this formula that $C_{x,x} = C_{y,y} = C_{z,z} = -1$ and $C_{x,x}^{f} = C_{y,y}^{f} = C_{z,z}^{f} = -1$, where $C_{i,j}^{f}$ is the spin-spin correlation parameter in the final state.

5.2.2 *The negative parity*

For $\pi = -1$ one has $S = 1$ in the reaction $pp \to \Sigma^+\Theta^+$. In this case the amplitude (1) can be written as the following sum of the two terms f_0 and f_1, describing the transitions ${}^3P_0 \to {}^1S_0$ and ${}^3P_1 \to {}^3S_1$, respectively,

$$M^{\mu_3\,\mu_4}_{\mu_1\,\mu_2} = f_0 + f_1, \tag{119}$$

where

$$f_0 = -\frac{1}{\sqrt{4\pi}} \sum_{\beta=x,y,z} (\chi^+_{\mu_1}\sigma_\beta \frac{i\sigma_y}{\sqrt{2}}\, \chi^{(T)+}_{\mu_2})\, (\chi^{(T)}_{\mu_3} \frac{-i\sigma_y}{\sqrt{2}}\, \chi_{\mu_4})\hat{k}_\beta\, a^1_0, \tag{120}$$

$$f_1 = i\sqrt{\frac{3}{8\pi}} \sum_{\alpha=x,y,z} \epsilon_{i\,l\,\alpha}\, \hat{k}_i(\chi^+_{\mu_1}\sigma_l \frac{i\sigma_y}{\sqrt{2}}\, \chi^{(T)+}_{\mu_2})\; (\chi^{(T)}_{\mu_3} \frac{-i\sigma_y}{\sqrt{2}}\, \sigma_\alpha\chi_{\mu_4})\, a^1_1; \tag{121}$$

here $\epsilon_{i\,l\,\alpha}$ is the fully antisymmetric tensor.

$$|f_0|^2 = \frac{1}{4\pi}|a^1_0|^2 \sum_{\beta\,,\beta'=x,y,z} \frac{1}{8} Sp\{\sigma_\beta\,(1 - \boldsymbol{\sigma}\cdot\mathbf{p}_2)\,\sigma_{\beta'}(1 + \boldsymbol{\sigma}\cdot\mathbf{p}_1)\} \times \\ \times\frac{1}{8}Sp\{(1 + \boldsymbol{\sigma}\cdot\mathbf{p}_4)\,(1 - \boldsymbol{\sigma}\cdot\mathbf{p}_3)\}, \tag{122}$$

$$|f_1|^2 = \frac{3}{8\pi}|a^1_1|^2 \sum_{i,i',l,l',\alpha\,,\alpha'=x,y,z} \epsilon_{i\,l\,\alpha}\,\epsilon_{i'\,l'\,\alpha'}\,\hat{k}_i\,\hat{k}_{i'} \times \\ \times\frac{1}{8}Sp\{\sigma_l\,(1 - \boldsymbol{\sigma}\cdot\mathbf{p}_2)\,\sigma_{l'}(1 + \boldsymbol{\sigma}\cdot\mathbf{p}_1)\} \times \\ \times\frac{1}{8}Sp\{\sigma_\alpha\,(1 + \boldsymbol{\sigma}\cdot\mathbf{p}_4)\,\sigma_{\alpha'}(1 - \boldsymbol{\sigma}\cdot\mathbf{p}_3)\}, \tag{123}$$

$$f^*_0\, f_1 = -i\frac{1}{4\pi}\sqrt{\frac{3}{2}}\, a^1_1\,(a^1_0)^* \sum_{i,l,\alpha\,,\beta=x,y,z} \epsilon_{i\,l\,\alpha}\,\hat{k}_\beta\,\hat{k}_i \times \\ \times\frac{1}{8}Sp\{\sigma_l\,(1 - \boldsymbol{\sigma}\cdot\mathbf{p}_2)\,\sigma_\beta(1 + \boldsymbol{\sigma}\cdot\mathbf{p}_1)\} \times \\ \times\frac{1}{8}Sp\{\sigma_\alpha\,(1 + \boldsymbol{\sigma}\cdot\mathbf{p}_4)\,(1 - \boldsymbol{\sigma}\cdot\mathbf{p}_3)\}, \tag{124}$$

$$f_0\, f_1^* = i\frac{1}{4\pi}\sqrt{\frac{3}{2}}\, a_0^1\,(a_1^1)^* \sum_{i,l,\alpha\,,\beta=x,y,z} \epsilon_{i\,l\,\alpha}\,\hat{k}_\beta\,\hat{k}_i \times$$
$$\frac{1}{8} Sp\{\sigma_\beta\,(1-\boldsymbol{\sigma}\cdot\mathbf{p}_2)\,\sigma_l(1+\boldsymbol{\sigma}\cdot\mathbf{p}_1)\} \times$$
$$\times\frac{1}{8} Sp\{\sigma_\alpha\,(1-\boldsymbol{\sigma}\cdot\mathbf{p}_3)\,(1+\boldsymbol{\sigma}\cdot\mathbf{p}_4)\}, \qquad (125)$$

Performing the traces in Eq. (110), one can find finally

$$d\sigma(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p}_3,\mathbf{p}_4) = \Phi(|f_0|^2+|f_1|^2+f_0\,f_1^*+f_0^*\,f_1) = \frac{1}{4}d\sigma_0\left\{1+\left(|a_0^1|^2+3|a_1^1|^2\right)^{-1}\times\right.$$
$$\times\left\{|a_0^1|^2[(\mathbf{p}_1\cdot\mathbf{p}_2)-(\mathbf{p}_3\cdot\mathbf{p}_4)] + (3|a_1^1|^2-2|a_0^1|^2)(\mathbf{p}_1\cdot\hat{\mathbf{k}})(\mathbf{p}_2\cdot\hat{\mathbf{k}}) + 3|a_1^1|^2(\mathbf{p}_3\cdot\hat{\mathbf{k}})(\mathbf{p}_4\cdot\hat{\mathbf{k}}) +\right.$$
$$+3|a_1^1|^2(\mathbf{p}_1\cdot\hat{\mathbf{k}}+\mathbf{p}_2\cdot\hat{\mathbf{k}})(\mathbf{p}_4\cdot\hat{\mathbf{k}}+\mathbf{p}_3\cdot\hat{\mathbf{k}}) +$$
$$+\sqrt{6}Re\, a_0^1 a_1^{1*}\Big((\mathbf{p}_1\cdot\hat{\mathbf{k}}+\mathbf{p}_2\cdot\hat{\mathbf{k}})(\mathbf{p}_4\cdot\hat{\mathbf{k}}-\mathbf{p}_3\cdot\hat{\mathbf{k}}) - (\mathbf{p}_1+\mathbf{p}_2)\cdot(\mathbf{p}_4-\mathbf{p}_3)\Big)$$
$$-\sqrt{6}\,Im\,a_0^1\,(a_1^1)^*\Bigg((\hat{\mathbf{k}}\cdot[\mathbf{p}_4\times\mathbf{p}_3])(\mathbf{p}_1\cdot\hat{\mathbf{k}}+\mathbf{p}_2\cdot\hat{\mathbf{k}}) -$$
$$-[\mathbf{p}_4\times\mathbf{p}_3]\cdot(\mathbf{p}_1+\mathbf{p}_2) - (\mathbf{p}_2\cdot\hat{\mathbf{k}})((\mathbf{p}_4-\mathbf{p}_3)\cdot[\hat{\mathbf{k}}\times\mathbf{p}_1]) -$$
$$-(\mathbf{p}_1\cdot\hat{\mathbf{k}})((\mathbf{p}_4-\mathbf{p}_3)\cdot[\hat{\mathbf{k}}\times\mathbf{p}_2])\Bigg) -$$
$$-\sqrt{6}\,Re\,a_0^1\,(a_1^1)^*\Bigg(([\mathbf{p}_3\times\mathbf{p}_4]\cdot[\hat{\mathbf{k}}\times\mathbf{p}_1])(\mathbf{p}_2\cdot\hat{\mathbf{k}}) + ([\mathbf{p}_3\times\mathbf{p}_4]\cdot[\hat{\mathbf{k}}\times\mathbf{p}_2])(\mathbf{p}_1\cdot\hat{\mathbf{k}})\Bigg) -$$
$$-\left(|a_0^1|^2+3|a_1^1|^2\right)(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{p}_3\cdot\mathbf{p}_4) - 3|a_1^1|^2\Big[(\mathbf{p}_1\cdot\mathbf{p}_2)(\mathbf{p}_3\cdot\hat{\mathbf{k}})(\mathbf{p}_4\cdot\hat{\mathbf{k}}) +$$
$$\left(2|a_0^1|^2+3|a_1^1|^2\right)(\mathbf{p}_1\cdot\hat{\mathbf{k}})(\mathbf{p}_2\cdot\hat{\mathbf{k}})(\mathbf{p}_3\cdot\mathbf{p}_4) +$$
$$\left.\left.+3|a_1^1|^2(\hat{\mathbf{k}}\cdot[\mathbf{p}_1\times\mathbf{p}_3])(\hat{\mathbf{k}}\cdot[\mathbf{p}_2\times\mathbf{p}_4]) + (\hat{\mathbf{k}}\cdot[\mathbf{p}_1\times\mathbf{p}_4])(\hat{\mathbf{k}}\cdot[\mathbf{p}_2\times\mathbf{p}_3])\Big]\right\}\right\}. \qquad (126)$$

Here the unpolarized cross section is given by

$$d\sigma_0 = \frac{\Phi}{16\pi}\Big\{|a_0^1|^2+3|a_1^1|^2\Big\}. \qquad (127)$$

One can see from Eq. (126) that terms with interference of the amplitudes a_1^1 and a_0^1 are antisymmetric under permutation of the indices 3 and 4. This follows from the fact that the amplitude f_1 given by Eq. (121) is symmetric while the amplitude f_0 in Eq. (120) is antisymmetric under permutation of the

indices of the nonidentical particles 3 and 4. Thus, one can find from the *4*-th line in Eq. (126) that $K_x^x(1,3) = K_x^x(2,3) = -K_x^x(1,4) = -K_x^x(2,4)$, where $K_i^j(l,m)$ describes the polarization transfer from the *l*-th to *m*-th particle. All results obtained in the previous sections for the coefficients K_i^j, $C_{i,j}$, $C_{i,j}^f$ and $K_{1\,i}^{1\,j,10}$ $(i,j = x,y,z)$ for $j_i = \frac{1}{2}$ $(i = 1,2,3,4)$ and $\pi = -1$ are contained in Eq.(126). For example, one can find from Eq. (126) that the coefficient in front of the product $p_{1\,x}\,p_{3\,y}\,p_{4\,z}$, i.e. the coefficient $2\sqrt{2}SpFF^{+}K_{1x}^{1y,10}$, coincides with that given by Eq.(97). The coefficients in front of the $p_{1\,y}\,p_{3\,y}$ and $p_{1\,z}\,p_{3\,z}$ coincide with K_y^y and K_z^z in Eqs. (28) and (29), respectively. Furthermore, similarly to Eqs. (116) we find from Eq.(126) the coefficients for the initial spin-spin correlation with induced polarization of the *3*-rd particle as follows

$$\bar{K}_{z,y}^{x} = -\bar{K}_{z,x}^{y} = \bar{K}_{y,z}^{x} = -\bar{K}_{x,z}^{y} = \frac{\sqrt{6}\,Im\,a_1^1 a_0^{1*}}{3|a_1^1|^2 + |a_0^1|^2}. \qquad (128)$$

These coefficients are equal to zero for the spin-singlet initial state (see Eq. (118)).

6 Discussion and conclusion

Our analysis is restricted by the threshold region, assuming the s-wave dominance in the relative motion of the final particles. This assumption is expected to be a reasonable approximation up to few MeV of the excess energy [37]. To see it one should mention that for a high momentum transfer, what is a specific feature of the reaction $NN \to Y\Theta^{+}$, in the near threshold regime the final partial wave with orbital momentum l contributes to the amplitude of the reaction as $\sim (p_f/Q)^l$, were p_f is the final c.m.s. momentum and Q denotes an intrinsic scale of the process which is determined by the transferred momentum. Therefore, the contribution of the higher partial waves $(l \neq 0)$ is suppressed at the threshold $(p_f < Q)$ as compared to $l = 0$. An additional suppression of the $l \neq 0$ partial waves can be provided by centrifugal barrier, if the final state short-range interaction between the hyperon Y and Θ^{+} is

strong enough.

When considering energy dependence of the spin observables near the threshold, one should note that if for a certain spin S (at given T and π) Eq.(3) holds at the threshold, then above the threshold for the same spin S one has the following relation:

$$(-1)^S = \pi\,(-1)^{T+1}(-1)^l. \tag{129}$$

Eqs. (3) and (129) show that only even orbital momenta l in the final state are allowed for this spin S. In contrast, for the other spin $\bar{S}$, which is not allowed at the threshold for the same T and π (for the s wave in the final state), only odd orbital momenta $\bar{l}$ contribute to the final state above the threshold. Thus, taking into account the above mentioned p_f^l-dependence of the transition amplitudes, one can see that (due to a p-wave contribution) the cross section ${}^{2\bar{S}+1}d\sigma_{\bar{M}}$ increases with the excess energy ($W \sim p_f^2$) in the near threshold region by one power of W faster as compared to the ${}^{2S+1}\,\mathrm{d}\sigma_M$ cross section. Here ${}^{2S+1}\,\mathrm{d}\sigma_M$ denotes the cross section of the reaction which is initiated in the NN spin state $|SM>$ [49]. For example, for the reaction $pp \to \Sigma^+\Theta^+$ one has ${}^3d\sigma_M/p_f \sim const(W)$ and ${}^1d\sigma_{M=0}/p_f \sim W$, if $\pi_\Theta = -1$. For $\pi_\Theta = +1$ the energy dependences of the singlet and triplet cross sections are interchanged. This difference allows one to determine the P-parity of the Θ^+ unambiguously by measurement of the energy dependence of the observables $d\sigma_0\, C_{j,j}$ in the near threshold region [55], because the unpolarized cross section $d\sigma_0$ can be separated into spin-singlet (${}^1\,\mathrm{d}\sigma_{M=0}$) and spin-triplet (${}^3d\sigma_M$) cross sections using the spin-spin correlation parameters $C_{x,x}$, $C_{y,y}$ and $C_{z,z}$ [49] [7]. Similar arguments can be found for energy dependence of the observables $d\sigma_0\, K_y^y$ [55], $d\sigma_0\, K_{J_N\,M_N}^{J_Y\,M_Y,\,J_\Theta\,M_\Theta}$ and $d\sigma_0\,\bar{K}_{x,\,z}^y$, although the singlet-to-triplet separation

[7] In particular, the sum $\Sigma = C_{x,x}+C_{y,y}+C_{z,z}$ (see Eq.(38)) is equal to $+1$ (-3) for the initial spin-singlet (triplet) state at any excess energy W independently on the reaction mechanism. Thus, a measurement of Σ determines the ratio of the triplet-singlet difference to the unpolarized cross section as $\delta = \frac{{}^3\sigma - {}^1\sigma}{\sigma_0} = \frac{1}{2}(1+\Sigma)$. This observable allows one to determine the P-parity of the Θ^+ in limiting cases $\delta = +1$ or $\delta = -1$, which are expected to occur near the threshold region.

cannot be applied here.

Obviously, quantitative results for spin observables above the threshold can be obtained only under additional assumptions about the dynamics of this reaction. In order to estimate the upper limit for the excess energy W for P-parity determination in the reaction $pp \to \Sigma^+\Theta^+$, the authors of Ref. [38] assumed the (reduced) partial wave amplitudes for different orbital momenta are comparable each with other in magnitude. Under this assumption they found that for excess energy less than 50 MeV the sign of the $C_{y,y}$ can be used for unambiguous determination of P-parity (for the case of $j_\Theta = \frac{1}{2}$). Later on this result was tested within a certain model of the $NN \to Y\Theta^+$ reaction in Ref. [55] and energy dependence of $d\sigma_0\, C_{j,j}$ and $d\sigma_0\, K_y^y$ was found to be more suitable for P-parity measurement within the same region of the excess energy $W < 50$ MeV.

Existing model calculations of the Θ^+ production in NN-collisions [53,54,55] are performed on the basis of the kaon exchanges in the Born approximation, while the mechanism of this reaction might be more complicated [56]. Furthermore, the initial and final state interactions were neglected in [53,54,55], although these can change the relative phases of the spin-amplitudes and therefore provide a significant effect of the polarization observables. Since at present neither the Θ^+ production mechanism nor the strength of the final state interaction are known and the spin of the Θ^+ is not measured, it is impossible to construct a quantitatively reliable model for the reaction in question. Therefore a general model-independent phenomenological approach for spin-observables at the threshold of the reaction $NN \to Y\Theta^+$ is still an appropriate method at the first step of analysis, whereas energy dependence of the observables in the near threshold region can be considered in a largely model independent way according to the method developed in Ref.[55].

The main idea of the work [37] for determination of the P-parity of the Θ^+ in the $\vec{p}\vec{p} \to \Sigma^+\Theta^+$ reaction is based on the fact that for $\pi_\Theta = +1$ (-1) only spin singlet (triplet) initial state is allowed at the threshold and therefore

the cross section is non-zero only for antiparallel (parallel) spins of the initial protons. This provides very clear signal for π_Θ. As it seen from Eqs.(8) and (15), this signal does not depend on the spins of the Y and Θ^+. This conclusion does not depend also on the isospin of the Θ^+ when it is equal to $0, 1$ and 2. Therefore, this reaction seems as a real tool for P-parity determination of the Θ^+. The problem is, however, connected with a complexity of such kind of experiments. The cross section near the threshold is suppressed by the phase space factor. Furthermore, the requirement of polarized beam and target could reduce the luminosity by two orders of magnitude as compared to unpolarized measurements. Thus, one has to explore others possibilities related to unpolarized or single-polarized experiments.

In the present analysis of the reaction $NN \to Y\Theta^+$ we use two such opportunities. (i) The vector polarization of the hyperon Y and (ii) tensor polarization of the Θ^+ (for $j_\Theta > \frac{1}{2}$) are measurable without performing a secondary scattering. So, the vector polarization of the hyperon Y can be measured via its weak decay $Y \to N + \pi$, because P-parity violation provides a large asymmetry in angular distribution of the decay products. The tensor polarization of the Θ^+, t_{J0}, for even ranks J can be measured by angular distribution in the strong decay $\Theta^+ \to N + K$. For example, according to Ref. [52], the angular distribution in the helicity frame, $I_{h.f.}(\theta)$, determines the following combinations of the spin-density matrix $\rho_{mm'}$ (m and m' are the spin projections) of the decaying spin-$\frac{3}{2}$ particle: $\rho_{\frac{1}{2},\frac{1}{2}} + \rho_{-\frac{1}{2},-\frac{1}{2}}$, $\rho_{\frac{3}{2},\frac{3}{2}} + \rho_{-\frac{3}{2},-\frac{3}{2}}$. It is easy to find that these combinations determine the t_{00} and t_{20}. A similar conclusion is valid for the spin $\frac{5}{2}$, where the additional combination $\rho_{\frac{5}{2},\frac{5}{2}} + \rho_{-\frac{5}{2},-\frac{5}{2}}$ is appeared that allows one to measure the tensor t_{40}. The angular distribution $I_{h.f.}(\theta)$ in the decay $\Theta^+ \to N + K$ is determined by the spin of the Θ^+. For the spin -$\frac{1}{2}$ decaying particle the angular distribution $I_{h.f.}(\theta)$ is isotropic [52]. Thus, measurement of angular distribution in the decay $\Theta^+ \to N + K$ allows one to determine the Θ^+ spin. In order to extract the tensor polarization t_{JM} with $M \neq 0$ one needs to know the longitudinal and transversal polarization of the decaying particle [52]. Such measurements require to perform secondary

rescatterings and, hence, are unrealistic for the reaction $NN \to Y\Theta^+$ and not considered here.

The main results of this paper related to the threshold kinematics of the $NN \to Y\Theta^+$ reaction can be summarized as follows.

Spin-spin correlation in the initial state $C_{i,j}$. Eqs. (8), (15) and (35) allow us to conclude that for $S = 1$ the spin-spin correlation coefficient $C_{y,y}$ is always non-negative in the reaction $1 + 2 \to 3 + 4$ at the threshold independently of the spins j_3 and j_4. On the contrary, for $S = 0$, the spin-spin correlation coefficients $C_{x,x} = C_{y,y} = C_{z,z}$ are equal to -1 for arbitrary spins of final particles. The obtained result allows one to determine unambiguously the P-parity of the Θ^+ by measurement of $C_{y,y}$ in the reaction $pp \to \Sigma^+\Theta^+$. The total isospin of this channel is fixed ($T = 1$) therefore the spin S of the initial nucleons is directly related to the P-parity π_Θ of the Θ^+ as $(-1)^S = \pi_\Theta$. In the reaction $pn \to \Lambda^0\Theta^+$ one has either $(-1)^S = -\pi_\Theta$, if the isospin of the Θ^+ is even ($I_\Theta = 0,\, 2$), or $(-1)^S = \pi_\Theta$, if $I_\Theta = 1$. Therefore, both the P-parity and the isospin of the Θ^+ can be determined unambiguously by combined measurement of $C_{y,y}$ in these two reactions.

Spin-spin correlation in the final state $C^f_{i,j}$, given by Eqs.(44), (46)-(58), can be used in principle for P-parity determination in a binary reaction $1+2 \to \vec{3}+\vec{4}$, if the vector polarizations of final particles are measurable in some way. In the reaction $NN \to \vec{Y}\vec{\Theta}^+$ only the polarization of the hyperon Y is self-analyzing via its weak decay, but a possibility to measure the polarization of the Θ^+ is very questionable. Non-zero spin-tensor correlation parameters in the final state $K_{00}^{10,J_\Theta 0}$ are allowed only for the odd rank J_Θ and, therefore, also unlikely can be measured in the $NN \to Y\Theta^+$ reaction.

The spin-transfer coefficient K_i^j. The vector polarization transfer is strongly correlated with the P-parity of the Θ^+ and the isospin of the NN-channel. The coefficient K_y^y is zero for the spin singlet and non-zero for the spin-triplet NN-state. In other words, in the reaction $pp \to \Sigma^+\Theta^+$ one has $K_y^y \neq 0$ for

$\pi_\Theta = -1$ and $K_y^y = 0$ for $\pi_\Theta = +1$. In the reaction $pn \to \Lambda\Theta^+$ the relation between K_y^y and π_Θ is inverted, if the Θ^+ is an isoscalar. For $j_\Theta = \frac{1}{2}$ we found $K_z^z \geq 0$ independently of π_Θ. For the isovector Θ^+, the spin observables in the reactions $pp \to \Sigma\Theta^+$ and $pn \to \Lambda\Theta^+$ are identical.

Tensor polarization of the Θ^+, t_{J0}, for even rank J can be measured by analysis of the angular distribution in the strong decay $\Theta \to N + K$ that does not require polarized beam and/or target. At given $j_\Theta > \frac{1}{2}$, as it seen from Eqs. (78), the coefficients $K_{00}^{\text{-}00,J0} \sim t_{J0}$ at $J = 0,\, 2,\, 4, \ldots$ are different for $\pi = -1$ and $\pi = +1$. The absolute value and the sign of this difference depend on the dynamics of the reaction $NN \to Y\Theta^+$. Hence, it is necessary to study these observables within definite mechanisms of the reaction. At last, the values of t_{J0} at even rank J give a definite restriction to the spin of the Θ^+.

The spin-transfer coefficient $K_{1y}^{1y,J0}$. If the spin of the Θ^+ is higher than $\frac{1}{2}$, then there are additional possibilities for P-parity determination. For even rank J the spin-tensor correlation induced by polarized beam, $K_{1y}^{1y,J0}$ and $K_{1z}^{1z,J0}$ are zero for the spin singlet and non-zero for the spin-triplet states. Therefore, similarly to the K_y^y coefficient, for given isospin and P-parity of the Θ^+, the coefficient $K_{1y}^{1y,J0}$ is non-zero only for one isospin channel of the reaction $NN \to Y\Theta^+$ and equals zero for another one. This statement is valid for any spin of the Θ^+. Therefore, measurements of both coefficients $K_{1y}^{1y,J0}$ and K_y^y in the reactions $pp \to \Sigma^+\Theta^+$ and $pn \to \Lambda\Theta^+$ give a strong test for the P-parity determination.

The spin-transfer coefficients $\bar{K}_{x,z}^y$ given by Eqs. (116) and (128) are non-zero only for the spin-triplet initial state. These coefficients are measurable in the reaction $\vec{N}\vec{N} \to \vec{Y}\Theta^+$ and, therefore, can be used as an additional test in the P-parity determination.

In conclusion, the formalism for double and triple spin correlation parameters of a binary reaction is derived at the threshold for arbitrary spins of the participating particles. For the spin-$\frac{1}{2}$ baryons the obtained formulae are checked

by different methods. The formalism can be applied for P-parity determination of the Θ^+ in the reaction $NN \to Y\Theta^+$ and any narrow resonance with arbitrary spin.

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Appendix

Here we derive Eqs. (23) and (24) for the spin-transfer coefficient K_i^j. From Eqs. (1), (18) and (22) we find

$$\begin{aligned} SpFF^+ K_\lambda^\kappa = &\sum_{\substack{L\,M_L\,S\,M_S\,J\,M_J\\ m_1\,m_2\,m_3\,m_4}} \sum_{\substack{L'\,M_L'\,S'\,M_S'\,J'\,M_J'\\ m_1'\,m_2'\,m_3'\,m_4'}} a_J^{LS}(a_{J'}^{L'\,S'})^*\, Y_{LM}^*(\mathbf{k}) Y_{L'M'}(\mathbf{k}) \times \\ &(j_1 m_1 j_2 m_2|S\,M_S)(j_3 m_3 j_4 m_4|J\,M_J)(SM_S\,LM_L|J\,M_J) \times \\ &\times (j_1 m_1' j_2 m_2'|S'\,M_S')(j_3 m_3' j_4 m_4'|J'\,M_J')(S'\,M_S'\,L'\,M_L'|J'\,M_J')\, Q_\lambda^\kappa, \end{aligned} \quad (A.1)$$

where Q_λ^κ is the following trace

$$\begin{aligned} Q_\lambda^\kappa = Sp\Big\{ &\chi_{j_1\,m_1}^+(1)\chi_{j_2\,m_2}^+(2)\chi_{j_3\,m_3}(3)\,\chi_{j_4\,m_4}(4) S_\lambda(1) \times \\ &\times \chi_{j_4\,m_4'}^+(4)\chi_{j_3\,m_3'}^+(3)\chi_{j_2\,m_2'}(2)\,\chi_{j_1\,m_1'}(1) S_\kappa(3)\Big\}. \end{aligned} \quad (A.2)$$

Using the following representation [51] for the spin operator $S_\lambda(j)$ of the kth particle ($k = 1, \ldots, 4$) with the spin j

$$S_\lambda(k) = \sqrt{j(j+1)} \sum_{m\,m'} (jm\,1\lambda|j\,m')\chi_{j\,m'}(k)\chi_{j\,m}^+(k) \quad (A.3)$$

and the relations

$$Sp\left\{\chi_{j\,m}^+(k)\chi_{j\,m}(k)\right\} = Sp\left\{\chi_{j\,m}(k)\chi_{j\,m}^+(k)\right\} = \delta_{m\,m'}, \quad (A.4)$$

one can find

$$Sp\left\{\chi^+_{j\,m'}(k)S_\lambda(k)\chi_{j\,m}(k)\right\} = Sp\left\{\chi_{j\,m}(k)\chi^+_{j\,m'}(k)S_\lambda(k)\right\} =$$
$$= (-1)^\lambda\sqrt{j(j+1)}\,(jm\,1-\lambda|j\,m'). \qquad \text{(A.5)}$$

Making use Eqs. (A.5) and (A.4), one can present Eq. (A.2) as

$$Q^\kappa_\lambda = (-1)^{\lambda+\kappa}\,\delta_{m_2\,m'_2}\,\delta_{m_4\,m'_4}\sqrt{j_1(j_1+1)j_3(j_3+1)}(j_3m'_3\,1-\kappa|j_3m_3)(j_1m_1\,1-\lambda|j_1\,m'_1). \qquad \text{(A.6)}$$

After substituting Eq. (A.6) into Eq. (A.1), one has to perform summation over the spin projections in Eq. (A.1). It can be done using the following relations

$$\sum_{m_1\,m'_1\,m_2}(j_1m_1\,j_2m_2|S\,M_S)(j_1m'_1\,j_2m_2|S'\,M'_S)(j_1m_11-\lambda|j_1m'_1) =$$
$$(-1)^{j_1+j_2+S+1}\sqrt{(2j_1+1)(2S+1)}(SM_S\,1\,-\lambda|S'\,M'_S)\begin{Bmatrix} j_1 & j_2 & S \\ S' & 1 & j_1\end{Bmatrix}, \qquad \text{(A.7)}$$
$$\sum_{m_3\,m'_3\,m_4}(j_3m_3\,j_4m_4|J\,M_J)(j_3m'_3\,j_3m_4|J'\,M'_J)(j_3m'_31-\kappa|j_3m_3) =$$
$$= (-1)^{j_3+j_4+J'+1}\sqrt{(2j_3+1)(2J'+1)}(J'M'_J\,1-\kappa|J\,M_J)\begin{Bmatrix} j_3 & j_4 & J' \\ J & 1 & j_3\end{Bmatrix}, \qquad \text{(A.8)}$$
$$\sum_{M_S\,M'_S\,M_J\,M'_J}(SM_S\,LM_L|JM_J)(S'M'_S\,L'M'_L|J'M'_J)(SM_S\,1\,-\lambda|S'M'_S)(J'M'_J\,1-\kappa|J\,M_J) =$$
$$= \sum_{J_0\,M_0}(-1)^{L-S-S'-M'_L-1}(2J+1)\sqrt{(2S'+1)(2J'+1)}\,\times$$
$$\times(L'\,-M'LM|J_0M_0)(1\,-\lambda\,1-\kappa|J_0\,M_0)\begin{Bmatrix} J & S & L \\ J' & S' & L' \\ 1 & 1 & J_0\end{Bmatrix} \qquad \text{(A.9)}$$

After that Eq. (A.1) can be written as Eq. (23).

In order to obtain Eqs. (24), we use here the relation between the Cartesian and spherical components of the spin operator [51]

$$
\begin{aligned}
S_{+1} &= -\frac{1}{\sqrt{2}}(S_x + iS_y), \\
S_{-1} &= \frac{1}{\sqrt{2}}(S_x - iS_y), \\
S_0 &= S_z.
\end{aligned} \tag{A.10}
$$

From Eqs. (22) and (A.10) one can find

$$
\begin{aligned}
K^{-1}_{+1} &= -\frac{1}{2}\left[K^x_x + K^y_y + iK^x_y - iK^y_x\right], \\
K^{+1}_{-1} &= -\frac{1}{2}\left[K^x_x + K^y_y - iK^x_y + iK^y_x\right], \\
K^{+1}_{+1} &= \frac{1}{2}\left[K^x_x - K^y_y + iK^x_y + iK^y_x\right], \\
K^{-1}_{-1} &= \frac{1}{2}\left[K^x_x - K^y_y - iK^x_y - iK^y_x\right].
\end{aligned} \tag{A.11}
$$

As was noted in the text after Eq. (23), the angular momentum J_0 is even. Therefore one has $(1-\lambda 1 \lambda | J_0 0) = (1 \lambda 1 - \lambda | J_0 0)$. It allows one to find from Eq. (23) the following relation

$$
K^{+1}_{-1} = K^{-1}_{+1}. \tag{A.12}
$$

Using Eqs.(23), (A.11) and (A.12) one finds $K^{-1}_{+1} - K^{+1}_{-1} = i(K^y_x - K^x_y) = 0$ and $K^{+1}_{+1} - {K_1}^{-1} = i(K^y_x + K^y_x) = 0$, therefore

$$
K^y_x = K^x_y = 0. \tag{A.13}
$$

Using relations $K^0_{-1} = K^0_{+1} = K^{+1}_0 = K^{-1}_0 = 0$, one can find

$$
K^z_x = K^z_y = K^x_z = K^y_z = 0. \tag{A.14}
$$

At last, taking into account the relation $K^{+1}_{+1} = 0$, which follows from Eq. (23), we find from the third equation in Eqs. (A.11)

$$
K^x_x = K^y_y. \tag{A.15}
$$

From Eqs. (A.15), (A.13) and (A.11) one can find Eqs. (24).

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